(* Asymptotic expected number of hairpins in saturated structures with theta=1 and p=1/2 *)

(*We first compute the dominant singularity and asymptotic number of saturated structures, using Drmota−Lalley−Woods Theorem. *)

```
Clear["*"]
Clear[p, S, D0, N0, z, R, eqn0, eqn, F, z0, y0, dFdzOfz0S0, d2FdyOfz0S0];
p = 1 / 2;
eqn = {S == D0 + N0, D0 == z + z^2, N0 == R D0 + p D0 z^2 + p N0 z^2 + p S D0 z^2 + p S N0 z^2,
  R == p D0 z^2 + p N0 z^2 + p R D0 z^2 + p R N0 z^2}
Eliminate[eqn, {N0, D0, R}]
F = (S^3 z^4 + S^2 z^2 (-4 + z^2) - z (4 + 4 z)) / (-1 (4 - 2 z^2))
```

```
NSolve[{F == S, D[F, S] == 1}, {z, S}];
z0 = 0.5343653037924583`
y0 = 2.1489053808896275`
```

```
dFdzOfz0S0 = D[F, z] /. {z → z0, S → y0}
```

```
d2FdyOfz0S0 = D[F, {S, 2}] /. {z → z0, S → y0}
c = Sqrt[z0 dFdzOfz0S0 / (2 Pi d2FdyOfz0S0)]
c * (1 / z0)^n n^(-3 / 2)
```

$$\left\{ S == D0 + N0, \; D0 == z + z^2, \; N0 == D0 \, R + \frac{D0 \, z^2}{2} + \frac{N0 \, z^2}{2} + \frac{1}{2} \, D0 \, S \, z^2 + \frac{1}{2} \, N0 \, S \, z^2, \right.$$

$$\left. R == \frac{D0 \, z^2}{2} + \frac{N0 \, z^2}{2} + \frac{1}{2} \, D0 \, R \, z^2 + \frac{1}{2} \, N0 \, R \, z^2 \right\}$$

$$S^3 z^4 + S \left(4 - 2 z^2\right) + S^2 z^2 \left(-4 + z^2\right) == z \left(4 + 4 z\right)$$

$$\frac{S^3 z^4 - z \left(4 + 4 z\right) + S^2 z^2 \left(-4 + z^2\right)}{-4 + 2 z^2}$$

0.534365

2.14891

6.92169

0.312055

1.37347

$$\frac{1.37347 \times 1.87138^n}{n^{3/2}}$$

(*Now, we compute mean,variance using Drmota's Theorem*)

```
Clear["*"]
Clear[p, rho, z0, y0, eqn, F, f, S, s, D0, N0, z, R, dfs, a, b]
p = 1 / 2;
eqn = {S == D0 + N0, D0 == z + z^2, N0 == R D0 +  p u D0 z^2 + p N0 z^2 + p u S D0 z^2 + p S N0 z^2,
    R == p u D0 z^2 + p N0 z^2 + p u R D0 z^2 + p R N0 z^2}


CellPrint["Eliminate all variables except S,u,z"]
Eliminate[eqn, {D0, N0, R}]
Collect[%, S, Simplify]
```

$$F = \left(S^3 z^4 + S^2 z^2 \left(-4 + z^2 + 2 (-1 + u) z^3 + 2 (-1 + u) z^4\right) + \right.$$
$$\left. z (1 + z) \left(-4 - 2 (-1 + u) z^2 + (-1 + u)^2 z^5 + (-1 + u)^2 z^6\right)\right) /$$
$$\left(- \left(4 - 2 z^2 - 4 (-1 + u) z^3 - 4 (-1 + u) z^4 + 2 (-1 + u) z^5 + \left(-1 + u^2\right) z^6 + 2 (-1 + u)^2 z^7 + (-1 + u)^2 z^8\right)\right)$$

```
f = (F /. S → s)
s - f
(* express over a common denominator*)
Together[s - f]
a = Numerator[%]

dfs = D[f, s]
1 - dfs
(* express over a common denominator*)
Together[1 - dfs]
(* a is numerator of s-f, and b is numerator of 1-dfs,
where both have identical denominators *)
b = Numerator[%]
(* a is numerator of s-f, and b is numerator of 1-dfs,
where both have identical denominators *)

If[Denominator[Together[s - f]] == Denominator[Together[1 - dfs]],
  CellPrint["Denominator of (s-f) same as that of (1-dfs)"],
  CellPrint["Denominator of (s-f) different than than of (1-dfs)"]]

(* NOTE: denominators of both expressions s-f and 1-dfs is the same *)
CellPrint["Now compute the resultant of numerators a,b to get relation between S,u,z"]

(* res =Resultant[s-f,1-dfs,s] *)


(*We compute resultant of numerators,
since we have Resultant[S-F,1-D[F,S],S]=0 which is not much use.*)
CellPrint["Express S-F and 1-D[F,S] as fractions over the same common denominator"]
CellPrint["Then compute the resultant of the numerators of these expressions"]
res = Resultant[a, b, s]
(* Replace z by z[u], a function of u *)


res /. z → z[u]
(* Now compute z'[u] *)
dres = D[%, u]
Simplify[Collect[dres, z'[u]]]
Solve[dres == 0, z'[u]]
dzu = Last[Last[Last[Solve[dres == 0, z'[u]]]]];
(*z[1] equals rho, the dominant singularity *)


rho = 0.5343653037924583`;
(* value of z0 in the first part of this file, the dominant singularity*)
dzuEvaluatedAt1 = (dzu /. u → 1) /. z[1] → rho
CellPrint[
```

```
"According to Drmota's Theorem 1, the mean equals -z'[1]/z[1], computed next. "]

mu = ((-dzu / z[u] /. u → 1) /. z[1] → rho)

(*Variance computation *)
(* d2zu is z''[1] *)
d2zu = (((D[dzu, u] /. u → 1) /. z[1] → rho) /. z'[1] → dzuEvaluatedAt1)
CellPrint["Now compute variance, which by Drmota is -z''[1]/z[1] + mu^2 + mu"]
var = -d2zu / rho + mu * mu + mu
```

$$\left\{ S == D0 + N0, \ D0 == z + z^2, \ N0 == D0\, R + \frac{N0\, z^2}{2} + \frac{1}{2}\, N0\, S\, z^2 + \frac{1}{2}\, D0\, u\, z^2 + \frac{1}{2}\, D0\, S\, u\, z^2, \right.$$

$$\left. R == \frac{N0\, z^2}{2} + \frac{1}{2}\, N0\, R\, z^2 + \frac{1}{2}\, D0\, u\, z^2 + \frac{1}{2}\, D0\, R\, u\, z^2 \right\}$$

Eliminate all variables except S,u,z

$$S^3\, z^4 + S^2\, z^2 \left(-4 + z^2 - 2\, z^3 + 2\, u\, z^3 - 2\, z^4 + 2\, u\, z^4 \right) +$$
$$S \left(4 - 2\, z^2 + 4\, z^3 - 4\, u\, z^3 + 4\, z^4 - 4\, u\, z^4 - 2\, z^5 + 2\, u\, z^5 - z^6 + \right.$$
$$\left. u^2\, z^6 + 2\, z^7 - 4\, u\, z^7 + 2\, u^2\, z^7 + z^8 - 2\, u\, z^8 + u^2\, z^8 \right) ==$$
$$z \left(4 + 4\, z - 2\, z^2 + 2\, u\, z^2 - 2\, z^3 + 2\, u\, z^3 - z^5 + 2\, u\, z^5 - u^2\, z^5 - 2\, z^6 + 4\, u\, z^6 - 2\, u^2\, z^6 - z^7 + 2\, u\, z^7 - u^2\, z^7 \right)$$

$$S^3\, z^4 + S^2\, z^2 \left(-4 + z^2 + 2\, (-1+u)\, z^3 + 2\, (-1+u)\, z^4 \right) +$$
$$S \left(4 - 2\, z^2 - 4\, (-1+u)\, z^3 - 4\, (-1+u)\, z^4 + 2\, (-1+u)\, z^5 + \left(-1+u^2 \right) z^6 + 2\, (-1+u)^2\, z^7 + (-1+u)^2\, z^8 \right) ==$$
$$-z\, (1+z) \left(-4 - 2\, (-1+u)\, z^2 + (-1+u)^2\, z^5 + (-1+u)^2\, z^6 \right)$$

$$\left(S^3\, z^4 + S^2\, z^2 \left(-4 + z^2 + 2\, (-1+u)\, z^3 + 2\, (-1+u)\, z^4 \right) + \right.$$
$$\left. z\, (1+z) \left(-4 - 2\, (-1+u)\, z^2 + (-1+u)^2\, z^5 + (-1+u)^2\, z^6 \right) \right) /$$
$$\left(-4 + 2\, z^2 + 4\, (-1+u)\, z^3 + 4\, (-1+u)\, z^4 - 2\, (-1+u)\, z^5 - \left(-1+u^2 \right) z^6 - 2\, (-1+u)^2\, z^7 - (-1+u)^2\, z^8 \right)$$

$$\left(s^3\, z^4 + s^2\, z^2 \left(-4 + z^2 + 2\, (-1+u)\, z^3 + 2\, (-1+u)\, z^4 \right) + \right.$$
$$\left. z\, (1+z) \left(-4 - 2\, (-1+u)\, z^2 + (-1+u)^2\, z^5 + (-1+u)^2\, z^6 \right) \right) /$$
$$\left(-4 + 2\, z^2 + 4\, (-1+u)\, z^3 + 4\, (-1+u)\, z^4 - 2\, (-1+u)\, z^5 - \left(-1+u^2 \right) z^6 - 2\, (-1+u)^2\, z^7 - (-1+u)^2\, z^8 \right)$$

$$s - \left(s^3\, z^4 + s^2\, z^2 \left(-4 + z^2 + 2\, (-1+u)\, z^3 + 2\, (-1+u)\, z^4 \right) + \right.$$
$$\left. z\, (1+z) \left(-4 - 2\, (-1+u)\, z^2 + (-1+u)^2\, z^5 + (-1+u)^2\, z^6 \right) \right) /$$
$$\left(-4 + 2\, z^2 + 4\, (-1+u)\, z^3 + 4\, (-1+u)\, z^4 - 2\, (-1+u)\, z^5 - \left(-1+u^2 \right) z^6 - 2\, (-1+u)^2\, z^7 - (-1+u)^2\, z^8 \right)$$

$$\left(4\, s - 4\, z - 4\, z^2 - 2\, s\, z^2 - 4\, s^2\, z^2 + 2\, z^3 + 4\, s\, z^3 - 2\, u\, z^3 - 4\, s\, u\, z^3 + 2\, z^4 + 4\, s\, z^4 + s^2\, z^4 + s^3\, z^4 - 2\, u\, z^4 - \right.$$
$$4\, s\, u\, z^4 - 2\, s\, z^5 - 2\, s^2\, z^5 + 2\, s\, u\, z^5 + 2\, s^2\, u\, z^5 + z^6 - s\, z^6 - 2\, s^2\, z^6 - 2\, u\, z^6 + 2\, s^2\, u\, z^6 + u^2\, z^6 + s\, u^2\, z^6 +$$
$$\left. 2\, z^7 + 2\, s\, z^7 - 4\, u\, z^7 - 4\, s\, u\, z^7 + 2\, u^2\, z^7 + 2\, s\, u^2\, z^7 + z^8 + s\, z^8 - 2\, u\, z^8 - 2\, s\, u\, z^8 + u^2\, z^8 + s\, u^2\, z^8 \right) /$$
$$\left(4 - 2\, z^2 + 4\, z^3 - 4\, u\, z^3 + 4\, z^4 - 4\, u\, z^4 - 2\, z^5 + 2\, u\, z^5 - z^6 + u^2\, z^6 + 2\, z^7 - 4\, u\, z^7 + 2\, u^2\, z^7 + z^8 - 2\, u\, z^8 + u^2\, z^8 \right)$$

$$4\, s - 4\, z - 4\, z^2 - 2\, s\, z^2 - 4\, s^2\, z^2 + 2\, z^3 + 4\, s\, z^3 - 2\, u\, z^3 - 4\, s\, u\, z^3 + 2\, z^4 + 4\, s\, z^4 + s^2\, z^4 + s^3\, z^4 - 2\, u\, z^4 -$$
$$4\, s\, u\, z^4 - 2\, s\, z^5 - 2\, s^2\, z^5 + 2\, s\, u\, z^5 + 2\, s^2\, u\, z^5 + z^6 - s\, z^6 - 2\, s^2\, z^6 - 2\, u\, z^6 + 2\, s^2\, u\, z^6 + u^2\, z^6 + s\, u^2\, z^6 +$$
$$2\, z^7 + 2\, s\, z^7 - 4\, u\, z^7 - 4\, s\, u\, z^7 + 2\, u^2\, z^7 + 2\, s\, u^2\, z^7 + z^8 + s\, z^8 - 2\, u\, z^8 - 2\, s\, u\, z^8 + u^2\, z^8 + s\, u^2\, z^8$$

$$\left(3\, s^2\, z^4 + 2\, s\, z^2 \left(-4 + z^2 + 2\, (-1+u)\, z^3 + 2\, (-1+u)\, z^4 \right) \right) /$$
$$\left(-4 + 2\, z^2 + 4\, (-1+u)\, z^3 + 4\, (-1+u)\, z^4 - 2\, (-1+u)\, z^5 - \left(-1+u^2 \right) z^6 - 2\, (-1+u)^2\, z^7 - (-1+u)^2\, z^8 \right)$$

$$1 - \left(3\, s^2\, z^4 + 2\, s\, z^2 \left(-4 + z^2 + 2\, (-1+u)\, z^3 + 2\, (-1+u)\, z^4 \right) \right) /$$
$$\left(-4 + 2\, z^2 + 4\, (-1+u)\, z^3 + 4\, (-1+u)\, z^4 - 2\, (-1+u)\, z^5 - \left(-1+u^2 \right) z^6 - 2\, (-1+u)^2\, z^7 - (-1+u)^2\, z^8 \right)$$

$$\left(4 - 2\,z^2 - 8\,s\,z^2 + 4\,z^3 - 4\,u\,z^3 + 4\,z^4 + 2\,s\,z^4 + 3\,s^2\,z^4 - 4\,u\,z^4 - 2\,z^5 - 4\,s\,z^5 + \right.$$
$$2\,u\,z^5 + 4\,s\,u\,z^5 - z^6 - 4\,s\,z^6 + 4\,s\,u\,z^6 + u^2\,z^6 + 2\,z^7 - 4\,u\,z^7 + 2\,u^2\,z^7 + z^8 - 2\,u\,z^8 + u^2\,z^8\Big) \Big/$$
$$\left(4 - 2\,z^2 + 4\,z^3 - 4\,u\,z^3 + 4\,z^4 - 4\,u\,z^4 - 2\,z^5 + 2\,u\,z^5 - z^6 + u^2\,z^6 + 2\,z^7 - 4\,u\,z^7 + 2\,u^2\,z^7 + z^8 - 2\,u\,z^8 + u^2\,z^8\right)$$

$$4 - 2\,z^2 - 8\,s\,z^2 + 4\,z^3 - 4\,u\,z^3 + 4\,z^4 + 2\,s\,z^4 + 3\,s^2\,z^4 - 4\,u\,z^4 - 2\,z^5 - 4\,s\,z^5 +$$
$$2\,u\,z^5 + 4\,s\,u\,z^5 - z^6 - 4\,s\,z^6 + 4\,s\,u\,z^6 + u^2\,z^6 + 2\,z^7 - 4\,u\,z^7 + 2\,u^2\,z^7 + z^8 - 2\,u\,z^8 + u^2\,z^8$$

Denominator of of (s-f) same as that of (1-dfs)

Now compute the resultant of numerators a,b to get relation between S,u,z

Express S-F and 1-D[F,S] as fractions over the same common denominator

Then compute the resultant of the numerators of these expressions

$$-128\,z^{11} - 144\,z^{12} + 96\,z^{13} + 352\,z^{14} + 192\,u\,z^{14} + 512\,z^{15} + 400\,u\,z^{15} + 220\,z^{16} + 256\,u\,z^{16} - 216\,z^{17} +$$
$$296\,u\,z^{17} - 96\,u^2\,z^{17} - 412\,z^{18} + 688\,u\,z^{18} - 292\,u^2\,z^{18} - 440\,z^{19} + 784\,u\,z^{19} - 344\,u^2\,z^{19} - 308\,z^{20} +$$
$$584\,u\,z^{20} - 292\,u^2\,z^{20} + 16\,u^3\,z^{20} - 208\,z^{21} + 480\,u\,z^{21} - 336\,u^2\,z^{21} + 64\,u^3\,z^{21} - 144\,z^{22} + 384\,u\,z^{22} -$$
$$336\,u^2\,z^{22} + 96\,u^3\,z^{22} - 64\,z^{23} + 192\,u\,z^{23} - 192\,u^2\,z^{23} + 64\,u^3\,z^{23} - 16\,z^{24} + 48\,u\,z^{24} - 48\,u^2\,z^{24} + 16\,u^3\,z^{24}$$

$$-128\,z[u]^{11} - 144\,z[u]^{12} + 96\,z[u]^{13} + 352\,z[u]^{14} + 192\,u\,z[u]^{14} + 512\,z[u]^{15} + 400\,u\,z[u]^{15} +$$
$$220\,z[u]^{16} + 256\,u\,z[u]^{16} - 216\,z[u]^{17} + 296\,u\,z[u]^{17} - 96\,u^2\,z[u]^{17} - 412\,z[u]^{18} +$$
$$688\,u\,z[u]^{18} - 292\,u^2\,z[u]^{18} - 440\,z[u]^{19} + 784\,u\,z[u]^{19} - 344\,u^2\,z[u]^{19} - 308\,z[u]^{20} +$$
$$584\,u\,z[u]^{20} - 292\,u^2\,z[u]^{20} + 16\,u^3\,z[u]^{20} - 208\,z[u]^{21} + 480\,u\,z[u]^{21} - 336\,u^2\,z[u]^{21} +$$
$$64\,u^3\,z[u]^{21} - 144\,z[u]^{22} + 384\,u\,z[u]^{22} - 336\,u^2\,z[u]^{22} + 96\,u^3\,z[u]^{22} - 64\,z[u]^{23} +$$
$$192\,u\,z[u]^{23} - 192\,u^2\,z[u]^{23} + 64\,u^3\,z[u]^{23} - 16\,z[u]^{24} + 48\,u\,z[u]^{24} - 48\,u^2\,z[u]^{24} + 16\,u^3\,z[u]^{24}$$

$$192\,z[u]^{14} + 400\,z[u]^{15} + 256\,z[u]^{16} + 296\,z[u]^{17} - 192\,u\,z[u]^{17} + 688\,z[u]^{18} - 584\,u\,z[u]^{18} + 784\,z[u]^{19} -$$
$$688\,u\,z[u]^{19} + 584\,z[u]^{20} - 584\,u\,z[u]^{20} + 48\,u^2\,z[u]^{20} + 480\,z[u]^{21} - 672\,u\,z[u]^{21} + 192\,u^2\,z[u]^{21} +$$
$$384\,z[u]^{22} - 672\,u\,z[u]^{22} + 288\,u^2\,z[u]^{22} + 192\,z[u]^{23} - 384\,u\,z[u]^{23} + 192\,u^2\,z[u]^{23} + 48\,z[u]^{24} -$$
$$96\,u\,z[u]^{24} + 48\,u^2\,z[u]^{24} - 1408\,z[u]^{10}\,\text{Derivative}[1][z][u] - 1728\,z[u]^{11}\,\text{Derivative}[1][z][u] +$$
$$1248\,z[u]^{12}\,\text{Derivative}[1][z][u] + 4928\,z[u]^{13}\,\text{Derivative}[1][z][u] +$$
$$2688\,u\,z[u]^{13}\,\text{Derivative}[1][z][u] + 7680\,z[u]^{14}\,\text{Derivative}[1][z][u] +$$
$$6000\,u\,z[u]^{14}\,\text{Derivative}[1][z][u] + 3520\,z[u]^{15}\,\text{Derivative}[1][z][u] +$$
$$4096\,u\,z[u]^{15}\,\text{Derivative}[1][z][u] - 3672\,z[u]^{16}\,\text{Derivative}[1][z][u] +$$
$$5032\,u\,z[u]^{16}\,\text{Derivative}[1][z][u] - 1632\,u^2\,z[u]^{16}\,\text{Derivative}[1][z][u] -$$
$$7416\,z[u]^{17}\,\text{Derivative}[1][z][u] + 12\,384\,u\,z[u]^{17}\,\text{Derivative}[1][z][u] -$$
$$5256\,u^2\,z[u]^{17}\,\text{Derivative}[1][z][u] - 8360\,z[u]^{18}\,\text{Derivative}[1][z][u] +$$
$$14\,896\,u\,z[u]^{18}\,\text{Derivative}[1][z][u] - 6536\,u^2\,z[u]^{18}\,\text{Derivative}[1][z][u] -$$
$$6160\,z[u]^{19}\,\text{Derivative}[1][z][u] + 11\,680\,u\,z[u]^{19}\,\text{Derivative}[1][z][u] -$$
$$5840\,u^2\,z[u]^{19}\,\text{Derivative}[1][z][u] + 320\,u^3\,z[u]^{19}\,\text{Derivative}[1][z][u] -$$
$$4368\,z[u]^{20}\,\text{Derivative}[1][z][u] + 10\,080\,u\,z[u]^{20}\,\text{Derivative}[1][z][u] -$$
$$7056\,u^2\,z[u]^{20}\,\text{Derivative}[1][z][u] + 1344\,u^3\,z[u]^{20}\,\text{Derivative}[1][z][u] -$$
$$3168\,z[u]^{21}\,\text{Derivative}[1][z][u] + 8448\,u\,z[u]^{21}\,\text{Derivative}[1][z][u] -$$
$$7392\,u^2\,z[u]^{21}\,\text{Derivative}[1][z][u] + 2112\,u^3\,z[u]^{21}\,\text{Derivative}[1][z][u] -$$
$$1472\,z[u]^{22}\,\text{Derivative}[1][z][u] + 4416\,u\,z[u]^{22}\,\text{Derivative}[1][z][u] -$$
$$4416\,u^2\,z[u]^{22}\,\text{Derivative}[1][z][u] + 1472\,u^3\,z[u]^{22}\,\text{Derivative}[1][z][u] -$$
$$384\,z[u]^{23}\,\text{Derivative}[1][z][u] + 1152\,u\,z[u]^{23}\,\text{Derivative}[1][z][u] -$$
$$1152\,u^2\,z[u]^{23}\,\text{Derivative}[1][z][u] + 384\,u^3\,z[u]^{23}\,\text{Derivative}[1][z][u]$$

$8 z[u]^{10} \left(6 (-1+u)^2 z[u]^{14} - 176 \text{ Derivative}[1][z][u] - 216 z[u] \text{ Derivative}[1][z][u] + \right.$
$156 z[u]^2 \text{ Derivative}[1][z][u] + 56 (11+6 u) z[u]^3 \text{ Derivative}[1][z][u] +$
$24 (-1+u)^2 z[u]^{13} (1+2 (-1+u) \text{ Derivative}[1][z][u]) +$
$4 (-1+u) z[u]^{12} \left(-12 + 9 u + 46 (-1+u)^2 \text{ Derivative}[1][z][u]\right) +$
$z[u]^{10} \left(73 - 73 u + 6 u^2 + 42 (-1+u)^2 (-13+4 u) \text{ Derivative}[1][z][u]\right) +$
$6 z[u]^4 (4 + 5 (32+25 u) \text{ Derivative}[1][z][u]) + z[u]^5 (50 + 8 (55+64 u) \text{ Derivative}[1][z][u]) +$
$12 (-1+u) z[u]^{11} \left(-5 + 2 u + 11 \left(3 - 5 u + 2 u^2\right) \text{ Derivative}[1][z][u]\right) +$
$z[u]^6 \left(32 - 17 \left(27 - 37 u + 12 u^2\right) \text{ Derivative}[1][z][u]\right) +$
$z[u]^8 \left(86 - 73 u - 19 \left(55 - 98 u + 43 u^2\right) \text{ Derivative}[1][z][u]\right) +$
$z[u]^7 \left(37 - 24 u - 9 \left(103 - 172 u + 73 u^2\right) \text{ Derivative}[1][z][u]\right) +$
$\left. 2 z[u]^9 \left(49 - 43 u + 5 \left(-77 + 146 u - 73 u^2 + 4 u^3\right) \text{ Derivative}[1][z][u]\right)\right)$

$\left\{\left\{\text{Derivative}[1][z][u] \rightarrow \right. \right.$
$\left(-24 z[u]^4 - 50 z[u]^5 - 32 z[u]^6 - 37 z[u]^7 + 24 u z[u]^7 - 86 z[u]^8 + 73 u z[u]^8 - \right.$
$98 z[u]^9 + 86 u z[u]^9 - 73 z[u]^{10} + 73 u z[u]^{10} - 6 u^2 z[u]^{10} - 60 z[u]^{11} +$
$84 u z[u]^{11} - 24 u^2 z[u]^{11} - 48 z[u]^{12} + 84 u z[u]^{12} - 36 u^2 z[u]^{12} -$
$\left. 24 z[u]^{13} + 48 u z[u]^{13} - 24 u^2 z[u]^{13} - 6 z[u]^{14} + 12 u z[u]^{14} - 6 u^2 z[u]^{14}\right) /$
$\left(-176 - 216 z[u] + 156 z[u]^2 + 616 z[u]^3 + 336 u z[u]^3 + 960 z[u]^4 + 750 u z[u]^4 + \right.$
$440 z[u]^5 + 512 u z[u]^5 - 459 z[u]^6 + 629 u z[u]^6 - 204 u^2 z[u]^6 - 927 z[u]^7 + 1548 u z[u]^7 -$
$657 u^2 z[u]^7 - 1045 z[u]^8 + 1862 u z[u]^8 - 817 u^2 z[u]^8 - 770 z[u]^9 + 1460 u z[u]^9 -$
$730 u^2 z[u]^9 + 40 u^3 z[u]^9 - 546 z[u]^{10} + 1260 u z[u]^{10} - 882 u^2 z[u]^{10} + 168 u^3 z[u]^{10} -$
$396 z[u]^{11} + 1056 u z[u]^{11} - 924 u^2 z[u]^{11} + 264 u^3 z[u]^{11} - 184 z[u]^{12} + 552 u z[u]^{12} -$
$\left. \left. \left. 552 u^2 z[u]^{12} + 184 u^3 z[u]^{12} - 48 z[u]^{13} + 144 u z[u]^{13} - 144 u^2 z[u]^{13} + 48 u^3 z[u]^{13}\right)\right\}\right\}$

-0.0664001

According to Drmota's Theorem 1, the mean equals -z'[1]/z[1], computed next.

0.12426

0.0564015

Now compute variance, which by Drmota is -z''[1]/z[1] + mu^2 + mu

0.0341517