

# (\* Asymptotic expected number of hairpins in saturated structures with theta=1 and p=3/8 \*)

(\*We first compute the dominant singularity and asymptotic number of saturated structures, using Drmota–Lalley–Woods Theorem. \*)

```
Clear["*"]
Clear[p, S, D0, N0, z, R, eqn0, eqn, F, z0, y0, dFdzOfz0S0, d2FdyOfz0S0];
p = 3 / 8;
eqn = {S == D0 + N0, D0 == z + z^2, N0 == R D0 + p D0 z^2 + p N0 z^2 + p S D0 z^2 + p S N0 z^2,
  R == p D0 z^2 + p N0 z^2 + p R D0 z^2 + p R N0 z^2}
Eliminate[eqn, {N0, D0, R}]
F = (9 S^3 z^4 + S^2 z^2 (-48 + 9 z^2) - z (64 + 64 z)) / (- (64 - 24 z^2))

NSolve[{F == S, D[F, S] == 1}, {z, S}];
z0 = 0.5864661087654325`
y0 = 2.4005719685352935`
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$$dFdzOfz0S0 = D[F, z] /. \{z \to z0, S \to y0\}$$

$$d2FdyOfz0S0 = D[F, \{S, 2\}] /. \{z \to z0, S \to y0\}$$

$$c = \text{Sqrt}[z0 dFdzOfz0S0 / (2 \text{Pi} d2FdyOfz0S0)]$$

$$c * (1 / z0)^n n^{(-3 / 2)}$$

$$\left\{ S == D0 + N0, D0 == z + z^2, N0 == D0 R + \frac{3 D0 z^2}{8} + \frac{3 N0 z^2}{8} + \frac{3}{8} D0 S z^2 + \frac{3}{8} N0 S z^2, \right.$$

$$\left. R == \frac{3 D0 z^2}{8} + \frac{3 N0 z^2}{8} + \frac{3}{8} D0 R z^2 + \frac{3}{8} N0 R z^2 \right\}$$

$$9 S^3 z^4 + S (64 - 24 z^2) + S^2 z^2 (-48 + 9 z^2) == z (64 + 64 z)$$

$$\frac{9 S^3 z^4 - z (64 + 64 z) + S^2 z^2 (-48 + 9 z^2)}{-64 + 24 z^2}$$

0.586466

2.40057

6.97437

0.279024

1.52744

$$\frac{1.52744 \times 1.70513^n}{n^{3/2}}$$

(\*Now, we compute mean,variance using Drmota's Theorem\*)

```

Clear["*"]
Clear[p, rho, z0, y0, eqn, F, f, S, s, D0, N0, z, R, dfs, a, b]
p = 3 / 8;
eqn = {S == D0 + N0, D0 == z + z^2, N0 == R D0 + p u D0 z^2 + p N0 z^2 + p u S D0 z^2 + p S N0 z^2,
  R == p u D0 z^2 + p N0 z^2 + p u R D0 z^2 + p R N0 z^2}

CellPrint["Eliminate all variables except S,u,z"]
Eliminate[eqn, {D0, N0, R}]
Collect[%, S, Simplify]
F = (9 S^3 z^4 + 3 S^2 z^2 (-16 + 3 z^2 + 6 (-1 + u) z^3 + 6 (-1 + u) z^4) +
  z (1 + z) (-64 - 24 (-1 + u) z^2 + 9 (-1 + u)^2 z^5 + 9 (-1 + u)^2 z^6)) /
  (- (64 - 24 z^2 - 48 (-1 + u) z^3 - 48 (-1 + u) z^4 + 18 (-1 + u) z^5 +
    9 (-1 + u)^2 z^6 + 18 (-1 + u)^2 z^7 + 9 (-1 + u)^2 z^8))

f = (F /. S -> s)
s - f
(* express over a common denominator*)
Together[s - f]
a = Numerator[%]

dfs = D[f, s]
1 - dfs
(* express over a common denominator*)
Together[1 - dfs]
(* a is numerator of s-f, and b is numerator of 1-dfs,
  where both have identical denominators *)
b = Numerator[%]
(* a is numerator of s-f, and b is numerator of 1-dfs,
  where both have identical denominators *)

If[Denominator[Together[s - f]] == Denominator[Together[1 - dfs]],
  CellPrint["Denominator of (s-f) same as that of (1-dfs)"],
  CellPrint["Denominator of (s-f) different than than of (1-dfs)"]]

(* NOTE: denominators of both expressions s-f and 1-dfs is the same *)
CellPrint["Now compute the resultant of numerators a,b to get relation between S,u,z"]

(* res =Resultant[s-f,1-dfs,s] *)

(*We compute resultant of numerators,
  since we have Resultant[S-F,1-D[F,S],S]=0 which is not much use.*)
CellPrint["Express S-F and 1-D[F,S] as fractions over the same common denominator"]
CellPrint["Then compute the resultant of the numerators of these expressions"]
res = Resultant[a, b, s]
(* Replace z by z[u], a function of u *)

res /. z -> z[u]
(* Now compute z'[u] *)
dres = D[%, u]
Simplify[Collect[dres, z'[u]]]
Solve[dres == 0, z'[u]]
dzu = Last[Last[Last[Solve[dres == 0, z'[u]]]]];
(*z[1] equals rho, the dominant singularity *)

rho = 0.5864661087654325`;
(* value of z0 in the first part of this file, the dominant singularity*)
dzuEvaluatedAt1 = (dzu /. u -> 1) /. z[1] -> rho
CellPrint[
  "According to Drmota's Theorem 1, the mean equals -z'[1]/z[1], computed next. "]

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mu = ((-dzu / z[u] /. u -> 1) /. z[1] -> rho)
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```
(*Variance computation *)
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```
(* d2zu is z''[1] *)
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```
d2zu = ((D[dzu, u] /. u -> 1) /. z[1] -> rho) /. z'[1] -> dzuEvaluatedAt1
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```
CellPrint["Now compute variance, which by Drmot is -z''[1]/z[1] + mu^2 + mu"]
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```
var = -d2zu / rho + mu * mu + mu
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$$\left\{ \begin{aligned} S &= D0 + N0, D0 = z + z^2, N0 = D0 R + \frac{3 N0 z^2}{8} + \frac{3}{8} N0 S z^2 + \frac{3}{8} D0 u z^2 + \frac{3}{8} D0 S u z^2, \\ R &= \frac{3 N0 z^2}{8} + \frac{3}{8} N0 R z^2 + \frac{3}{8} D0 u z^2 + \frac{3}{8} D0 R u z^2 \end{aligned} \right\}$$

Eliminate all variables except S,u,z

$$\begin{aligned} & 9 S^3 z^4 + S^2 z^2 (-48 + 9 z^2 - 18 z^3 + 18 u z^3 - 18 z^4 + 18 u z^4) + \\ & S (64 - 24 z^2 + 48 z^3 - 48 u z^3 + 48 z^4 - 48 u z^4 - 18 z^5 + 18 u z^5 - \\ & 9 z^6 + 9 u^2 z^6 + 18 z^7 - 36 u z^7 + 18 u^2 z^7 + 9 z^8 - 18 u z^8 + 9 u^2 z^8) = \\ & z (64 + 64 z - 24 z^2 + 24 u z^2 - 24 z^3 + 24 u z^3 - 9 z^5 + 18 u z^5 - 9 u^2 z^5 - 18 z^6 + \\ & 36 u z^6 - 18 u^2 z^6 - 9 z^7 + 18 u z^7 - 9 u^2 z^7) \\ & 9 S^3 z^4 + 3 S^2 z^2 (-16 + 3 z^2 + 6 (-1 + u) z^3 + 6 (-1 + u) z^4) + \\ & S (64 - 24 z^2 - 48 (-1 + u) z^3 - 48 (-1 + u) z^4 + 18 (-1 + u) z^5 + \\ & 9 (-1 + u)^2 z^6 + 18 (-1 + u)^2 z^7 + 9 (-1 + u)^2 z^8) = \\ & -z (1 + z) (-64 - 24 (-1 + u) z^2 + 9 (-1 + u)^2 z^5 + 9 (-1 + u)^2 z^6) \\ & (9 S^3 z^4 + 3 S^2 z^2 (-16 + 3 z^2 + 6 (-1 + u) z^3 + 6 (-1 + u) z^4) + \\ & z (1 + z) (-64 - 24 (-1 + u) z^2 + 9 (-1 + u)^2 z^5 + 9 (-1 + u)^2 z^6)) / \\ & (-64 + 24 z^2 + 48 (-1 + u) z^3 + 48 (-1 + u) z^4 - 18 (-1 + u) z^5 - \\ & 9 (-1 + u)^2 z^6 - 18 (-1 + u)^2 z^7 - 9 (-1 + u)^2 z^8) \\ & (9 S^3 z^4 + 3 S^2 z^2 (-16 + 3 z^2 + 6 (-1 + u) z^3 + 6 (-1 + u) z^4) + \\ & z (1 + z) (-64 - 24 (-1 + u) z^2 + 9 (-1 + u)^2 z^5 + 9 (-1 + u)^2 z^6)) / \\ & (-64 + 24 z^2 + 48 (-1 + u) z^3 + 48 (-1 + u) z^4 - 18 (-1 + u) z^5 - \\ & 9 (-1 + u)^2 z^6 - 18 (-1 + u)^2 z^7 - 9 (-1 + u)^2 z^8) \\ & s - (9 S^3 z^4 + 3 S^2 z^2 (-16 + 3 z^2 + 6 (-1 + u) z^3 + 6 (-1 + u) z^4) + \\ & z (1 + z) (-64 - 24 (-1 + u) z^2 + 9 (-1 + u)^2 z^5 + 9 (-1 + u)^2 z^6)) / (-64 + 24 z^2 + \\ & 48 (-1 + u) z^3 + 48 (-1 + u) z^4 - 18 (-1 + u) z^5 - 9 (-1 + u)^2 z^6 - 18 (-1 + u)^2 z^7 - 9 (-1 + u)^2 z^8) \\ & (64 s - 64 z - 64 z^2 - 24 s z^2 - 48 s^2 z^2 + 24 z^3 + 48 s z^3 - 24 u z^3 - 48 s u z^3 + 24 z^4 + \\ & 48 s z^4 + 9 s^2 z^4 + 9 s^3 z^4 - 24 u z^4 - 48 s u z^4 - 18 s z^5 - 18 s^2 z^5 + 18 s u z^5 + 18 s^2 u z^5 + \\ & 9 z^6 - 9 s z^6 - 18 s^2 z^6 - 18 u z^6 + 18 s^2 u z^6 + 9 u^2 z^6 + 9 s u^2 z^6 + 18 z^7 + 18 s z^7 - 36 u z^7 - \\ & 36 s u z^7 + 18 u^2 z^7 + 18 s u^2 z^7 + 9 z^8 + 9 s z^8 - 18 u z^8 - 18 s u z^8 + 9 u^2 z^8 + 9 s u^2 z^8) / \\ & (64 - 24 z^2 + 48 z^3 - 48 u z^3 + 48 z^4 - 48 u z^4 - 18 z^5 + 18 u z^5 - 9 z^6 + \\ & 9 u^2 z^6 + 18 z^7 - 36 u z^7 + 18 u^2 z^7 + 9 z^8 - 18 u z^8 + 9 u^2 z^8) \\ & 64 s - 64 z - 64 z^2 - 24 s z^2 - 48 s^2 z^2 + 24 z^3 + 48 s z^3 - 24 u z^3 - 48 s u z^3 + 24 z^4 + \\ & 48 s z^4 + 9 s^2 z^4 + 9 s^3 z^4 - 24 u z^4 - 48 s u z^4 - 18 s z^5 - 18 s^2 z^5 + 18 s u z^5 + 18 s^2 u z^5 + \\ & 9 z^6 - 9 s z^6 - 18 s^2 z^6 - 18 u z^6 + 18 s^2 u z^6 + 9 u^2 z^6 + 9 s u^2 z^6 + 18 z^7 + 18 s z^7 - 36 u z^7 - \\ & 36 s u z^7 + 18 u^2 z^7 + 18 s u^2 z^7 + 9 z^8 + 9 s z^8 - 18 u z^8 - 18 s u z^8 + 9 u^2 z^8 + 9 s u^2 z^8 \end{aligned}$$

$$\begin{aligned}
& \left( 27 s^2 z^4 + 6 s z^2 (-16 + 3 z^2 + 6(-1+u)z^3 + 6(-1+u)z^4) \right) / \\
& \left( -64 + 24 z^2 + 48(-1+u)z^3 + 48(-1+u)z^4 - \right. \\
& \quad \left. 18(-1+u)z^5 - 9(-1+u^2)z^6 - 18(-1+u)^2 z^7 - 9(-1+u)^2 z^8 \right) \\
& 1 - \left( 27 s^2 z^4 + 6 s z^2 (-16 + 3 z^2 + 6(-1+u)z^3 + 6(-1+u)z^4) \right) / \left( -64 + 24 z^2 + 48(-1+u)z^3 + \right. \\
& \quad \left. 48(-1+u)z^4 - 18(-1+u)z^5 - 9(-1+u^2)z^6 - 18(-1+u)^2 z^7 - 9(-1+u)^2 z^8 \right) \\
& \left( 64 - 24 z^2 - 96 s z^2 + 48 z^3 - 48 u z^3 + 48 z^4 + 18 s z^4 + 27 s^2 z^4 - 48 u z^4 - 18 z^5 - 36 s z^5 + 18 u z^5 + \right. \\
& \quad \left. 36 s u z^5 - 9 z^6 - 36 s z^6 + 36 s u z^6 + 9 u^2 z^6 + 18 z^7 - 36 u z^7 + 18 u^2 z^7 + 9 z^8 - 18 u z^8 + 9 u^2 z^8 \right) / \\
& \left( 64 - 24 z^2 + 48 z^3 - 48 u z^3 + 48 z^4 - 48 u z^4 - 18 z^5 + 18 u z^5 - 9 z^6 + \right. \\
& \quad \left. 9 u^2 z^6 + 18 z^7 - 36 u z^7 + 18 u^2 z^7 + 9 z^8 - 18 u z^8 + 9 u^2 z^8 \right) \\
& 64 - 24 z^2 - 96 s z^2 + 48 z^3 - 48 u z^3 + 48 z^4 + 18 s z^4 + 27 s^2 z^4 - 48 u z^4 - 18 z^5 - 36 s z^5 + 18 u z^5 + \\
& \quad 36 s u z^5 - 9 z^6 - 36 s z^6 + 36 s u z^6 + 9 u^2 z^6 + 18 z^7 - 36 u z^7 + 18 u^2 z^7 + 9 z^8 - 18 u z^8 + 9 u^2 z^8
\end{aligned}$$

Denominator of (s-f) same as that of (1-dfs)

Now compute the resultant of numerators a,b to get relation between S,u,z

Express S-F and 1-D[F,S] as fractions over the same common denominator

Then compute the resultant of the numerators of these expressions

$$\begin{aligned}
& -31850496 z^{11} - 34836480 z^{12} + 17915904 z^{13} + 64945152 z^{14} + 35831808 u z^{14} + 94058496 z^{15} + \\
& 73903104 u z^{15} + 42130368 z^{16} + 44789760 u z^{16} - 29393280 z^{17} + 41150592 u z^{17} - \\
& 13436928 u^2 z^{17} - 55007424 z^{18} + 94058496 u z^{18} - 40730688 u^2 z^{18} - 56267136 z^{19} + \\
& 102456576 u z^{19} - 46189440 u^2 z^{19} - 35691840 z^{20} + 68024448 u z^{20} - 34012224 u^2 z^{20} + \\
& 1679616 u^3 z^{20} - 21835008 z^{21} + 50388480 u z^{21} - 35271936 u^2 z^{21} + 6718464 u^3 z^{21} - 15116544 z^{22} + \\
& 40310784 u z^{22} - 35271936 u^2 z^{22} + 10077696 u^3 z^{22} - 6718464 z^{23} + 20155392 u z^{23} - \\
& 20155392 u^2 z^{23} + 6718464 u^3 z^{23} - 1679616 z^{24} + 5038848 u z^{24} - 5038848 u^2 z^{24} + 1679616 u^3 z^{24} \\
& -31850496 z[u]^{11} - 34836480 z[u]^{12} + 17915904 z[u]^{13} + 64945152 z[u]^{14} + 35831808 u z[u]^{14} + \\
& 94058496 z[u]^{15} + 73903104 u z[u]^{15} + 42130368 z[u]^{16} + 44789760 u z[u]^{16} - \\
& 29393280 z[u]^{17} + 41150592 u z[u]^{17} - 13436928 u^2 z[u]^{17} - 55007424 z[u]^{18} + \\
& 94058496 u z[u]^{18} - 40730688 u^2 z[u]^{18} - 56267136 z[u]^{19} + 102456576 u z[u]^{19} - \\
& 46189440 u^2 z[u]^{19} - 35691840 z[u]^{20} + 68024448 u z[u]^{20} - 34012224 u^2 z[u]^{20} + \\
& 1679616 u^3 z[u]^{20} - 21835008 z[u]^{21} + 50388480 u z[u]^{21} - 35271936 u^2 z[u]^{21} + \\
& 6718464 u^3 z[u]^{21} - 15116544 z[u]^{22} + 40310784 u z[u]^{22} - 35271936 u^2 z[u]^{22} + \\
& 10077696 u^3 z[u]^{22} - 6718464 z[u]^{23} + 20155392 u z[u]^{23} - 20155392 u^2 z[u]^{23} + \\
& 6718464 u^3 z[u]^{23} - 1679616 z[u]^{24} + 5038848 u z[u]^{24} - 5038848 u^2 z[u]^{24} + 1679616 u^3 z[u]^{24}
\end{aligned}$$

$$\begin{aligned}
 & 35\,831\,808 z[u]^{14} + 73\,903\,104 z[u]^{15} + 44\,789\,760 z[u]^{16} + 41\,150\,592 z[u]^{17} - 26\,873\,856 u z[u]^{17} + \\
 & 94\,058\,496 z[u]^{18} - 81\,461\,376 u z[u]^{18} + 102\,456\,576 z[u]^{19} - 92\,378\,880 u z[u]^{19} + 68\,024\,448 z[u]^{20} - \\
 & 68\,024\,448 u z[u]^{20} + 5\,038\,848 u^2 z[u]^{20} + 50\,388\,480 z[u]^{21} - 70\,543\,872 u z[u]^{21} + \\
 & 20\,155\,392 u^2 z[u]^{21} + 40\,310\,784 z[u]^{22} - 70\,543\,872 u z[u]^{22} + 30\,233\,088 u^2 z[u]^{22} + 20\,155\,392 z[u]^{23} - \\
 & 40\,310\,784 u z[u]^{23} + 20\,155\,392 u^2 z[u]^{23} + 5\,038\,848 z[u]^{24} - 10\,077\,696 u z[u]^{24} + 5\,038\,848 u^2 z[u]^{24} - \\
 & 350\,355\,456 z[u]^{10} \text{Derivative}[1][z][u] - 418\,037\,760 z[u]^{11} \text{Derivative}[1][z][u] + \\
 & 232\,906\,752 z[u]^{12} \text{Derivative}[1][z][u] + 909\,232\,128 z[u]^{13} \text{Derivative}[1][z][u] + \\
 & 501\,645\,312 u z[u]^{13} \text{Derivative}[1][z][u] + 1\,410\,877\,440 z[u]^{14} \text{Derivative}[1][z][u] + \\
 & 1\,108\,546\,560 u z[u]^{14} \text{Derivative}[1][z][u] + 674\,085\,888 z[u]^{15} \text{Derivative}[1][z][u] + \\
 & 716\,636\,160 u z[u]^{15} \text{Derivative}[1][z][u] - 499\,685\,760 z[u]^{16} \text{Derivative}[1][z][u] + \\
 & 699\,560\,064 u z[u]^{16} \text{Derivative}[1][z][u] - 228\,427\,776 u^2 z[u]^{16} \text{Derivative}[1][z][u] - \\
 & 990\,133\,632 z[u]^{17} \text{Derivative}[1][z][u] + 1\,693\,052\,928 u z[u]^{17} \text{Derivative}[1][z][u] - \\
 & 733\,152\,384 u^2 z[u]^{17} \text{Derivative}[1][z][u] - 1\,069\,075\,584 z[u]^{18} \text{Derivative}[1][z][u] + \\
 & 1\,946\,674\,944 u z[u]^{18} \text{Derivative}[1][z][u] - 877\,599\,360 u^2 z[u]^{18} \text{Derivative}[1][z][u] - \\
 & 713\,836\,800 z[u]^{19} \text{Derivative}[1][z][u] + 1\,360\,488\,960 u z[u]^{19} \text{Derivative}[1][z][u] - \\
 & 680\,244\,480 u^2 z[u]^{19} \text{Derivative}[1][z][u] + 33\,592\,320 u^3 z[u]^{19} \text{Derivative}[1][z][u] - \\
 & 458\,535\,168 z[u]^{20} \text{Derivative}[1][z][u] + 1\,058\,158\,080 u z[u]^{20} \text{Derivative}[1][z][u] - \\
 & 740\,710\,656 u^2 z[u]^{20} \text{Derivative}[1][z][u] + 141\,087\,744 u^3 z[u]^{20} \text{Derivative}[1][z][u] - \\
 & 332\,563\,968 z[u]^{21} \text{Derivative}[1][z][u] + 886\,837\,248 u z[u]^{21} \text{Derivative}[1][z][u] - \\
 & 775\,982\,592 u^2 z[u]^{21} \text{Derivative}[1][z][u] + 221\,709\,312 u^3 z[u]^{21} \text{Derivative}[1][z][u] - \\
 & 154\,524\,672 z[u]^{22} \text{Derivative}[1][z][u] + 463\,574\,016 u z[u]^{22} \text{Derivative}[1][z][u] - \\
 & 463\,574\,016 u^2 z[u]^{22} \text{Derivative}[1][z][u] + 154\,524\,672 u^3 z[u]^{22} \text{Derivative}[1][z][u] - \\
 & 40\,310\,784 z[u]^{23} \text{Derivative}[1][z][u] + 120\,932\,352 u z[u]^{23} \text{Derivative}[1][z][u] - \\
 & 120\,932\,352 u^2 z[u]^{23} \text{Derivative}[1][z][u] + 40\,310\,784 u^3 z[u]^{23} \text{Derivative}[1][z][u] \\
 & 31\,104 z[u]^{10} (162 (-1 + u)^2 z[u]^{14} - 11\,264 \text{Derivative}[1][z][u] - 13\,440 z[u] \text{Derivative}[1][z][u] + \\
 & 7488 z[u]^2 \text{Derivative}[1][z][u] + 1008 (29 + 16 u) z[u]^3 \text{Derivative}[1][z][u] + \\
 & 648 (-1 + u)^2 z[u]^{13} (1 + 2 (-1 + u) \text{Derivative}[1][z][u]) + \\
 & 108 (-1 + u) z[u]^{12} (-12 + 9 u + 46 (-1 + u)^2 \text{Derivative}[1][z][u]) + \\
 & 81 z[u]^{10} (27 - 27 u + 2 u^2 + 14 (-1 + u)^2 (-13 + 4 u) \text{Derivative}[1][z][u]) + \\
 & 72 z[u]^4 (16 + 45 (14 + 11 u) \text{Derivative}[1][z][u]) + \\
 & 72 z[u]^5 (33 + (301 + 320 u) \text{Derivative}[1][z][u]) + \\
 & 324 (-1 + u) z[u]^{11} (-5 + 2 u + 11 (3 - 5 u + 2 u^2) \text{Derivative}[1][z][u]) - \\
 & 9 z[u]^6 (-160 + 51 (35 - 49 u + 16 u^2) \text{Derivative}[1][z][u]) - \\
 & 27 z[u]^8 (-112 + 97 u + 19 (67 - 122 u + 55 u^2) \text{Derivative}[1][z][u]) - \\
 & 27 z[u]^7 (-49 + 32 u + 9 (131 - 224 u + 97 u^2) \text{Derivative}[1][z][u]) + \\
 & 54 z[u]^9 (61 - 55 u + 5 (-85 + 162 u - 81 u^2 + 4 u^3) \text{Derivative}[1][z][u])) \\
 & \{ \{ \text{Derivative}[1][z][u] \rightarrow \\
 & - (9 (128 z[u]^4 + 264 z[u]^5 + 160 z[u]^6 + 147 z[u]^7 - 96 u z[u]^7 + 336 z[u]^8 - 291 u z[u]^8 + \\
 & 366 z[u]^9 - 330 u z[u]^9 + 243 z[u]^{10} - 243 u z[u]^{10} + 18 u^2 z[u]^{10} + 180 z[u]^{11} - \\
 & 252 u z[u]^{11} + 72 u^2 z[u]^{11} + 144 z[u]^{12} - 252 u z[u]^{12} + 108 u^2 z[u]^{12} + \\
 & 72 z[u]^{13} - 144 u z[u]^{13} + 72 u^2 z[u]^{13} + 18 z[u]^{14} - 36 u z[u]^{14} + 18 u^2 z[u]^{14})) / \\
 & (-11\,264 - 13\,440 z[u] + 7488 z[u]^2 + 29\,232 z[u]^3 + 16\,128 u z[u]^3 + 45\,360 z[u]^4 + \\
 & 35\,640 u z[u]^4 + 21\,672 z[u]^5 + 23\,040 u z[u]^5 - 16\,065 z[u]^6 + 22\,491 u z[u]^6 - 7344 u^2 z[u]^6 - \\
 & 31\,833 z[u]^7 + 54\,432 u z[u]^7 - 23\,571 u^2 z[u]^7 - 34\,371 z[u]^8 + 62\,586 u z[u]^8 - 28\,215 u^2 z[u]^8 - \\
 & 22\,950 z[u]^9 + 43\,740 u z[u]^9 - 21\,870 u^2 z[u]^9 + 1080 u^3 z[u]^9 - 14\,742 z[u]^{10} + \\
 & 34\,020 u z[u]^{10} - 23\,814 u^2 z[u]^{10} + 4536 u^3 z[u]^{10} - 10\,692 z[u]^{11} + 28\,512 u z[u]^{11} - \\
 & 24\,948 u^2 z[u]^{11} + 7128 u^3 z[u]^{11} - 4968 z[u]^{12} + 14\,904 u z[u]^{12} - 14\,904 u^2 z[u]^{12} + \\
 & 4968 u^3 z[u]^{12} - 1296 z[u]^{13} + 3888 u z[u]^{13} - 3888 u^2 z[u]^{13} + 1296 u^3 z[u]^{13}) \} \} \\
 & -0.0729984
 \end{aligned}$$

According to Drmota's Theorem 1, the mean equals -z'[1]/z[1], computed next.

0.124472

0.0620853

Now compute variance, which by Drmota is  $-z''[1]/z[1] + \mu^2 + \mu$

0.0341015