

# (\* Asymptotic expected number of hairpins in saturated structures with theta=3 and p=1\*)

(\*We first compute the dominant singularity and asymptotic number of saturated structures, using Drmota–Lalley–Woods Theorem. \*)

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Clear["*"]
Clear[p, S, D0, N0, z, R, eqn0, eqn, F, z0, y0, dFdzoFz0S0, d2FdyOfz0S0];
p = 1;
eqn = {S == D0 + N0, D0 == z + z^2 + z^3 + z^4,
  N0 == R D0 + p (z^3 + z^4) z^2 + p N0 z^2 + p S (z^3 + z^4) z^2 + p S N0 z^2,
  R == p (z^3 + z^4) z^2 + p N0 z^2 + p R (z^3 + z^4) z^2 + p R N0 z^2}
Eliminate[eqn, {N0, D0, R}]
F = (S^3 z^4 + S^2 z^2 (-2 + z^2 - 2 z^3 - 2 z^4) - z (1 + z - z^5 - 2 z^6 - z^7)) /
  (- (1 - z^2 + 2 z^3 + 2 z^4 - 2 z^5 - z^6 + 2 z^7 + z^8))
NSolve[{F == S, D[F, S] == 1}, {z, S}];
z0 = 0.47550096719066426`
y0 = 1.4417998776529741`

dFdzoFz0S0 = D[F, z] /. {z -> z0, S -> y0}

d2FdyOfz0S0 = D[F, {S, 2}] /. {z -> z0, S -> y0}
c = Sqrt[z0 dFdzoFz0S0 / (2 Pi d2FdyOfz0S0)]
c * (1 / z0) ^ n n ^ (-3 / 2)

{S == D0 + N0, D0 == z + z^2 + z^3 + z^4, N0 == D0 R + N0 z^2 + N0 S z^2 + z^2 (z^3 + z^4) + S z^2 (z^3 + z^4),
  R == N0 z^2 + N0 R z^2 + z^2 (z^3 + z^4) + R z^2 (z^3 + z^4)}

S^3 z^4 + S^2 z^2 (-2 + z^2 - 2 z^3 - 2 z^4) + S (1 - z^2 + 2 z^3 + 2 z^4 - 2 z^5 - z^6 + 2 z^7 + z^8) == z (1 + z - z^5 - 2 z^6 - z^7)

S^3 z^4 + S^2 z^2 (-2 + z^2 - 2 z^3 - 2 z^4) - z (1 + z - z^5 - 2 z^6 - z^7)
-----
-1 + z^2 - 2 z^3 - 2 z^4 + 2 z^5 + z^6 - 2 z^7 - z^8

0.475501
1.4418
3.70018
0.481896
0.762291
0.762291 * 2.10305^n
-----
n^3/2

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(\*Now, we compute mean, variance using Drmota's Theorem\*)

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Clear["*"]
p = 1;
eqn = {S == D0 + N0, D0 == z + z^2 + z^3 + z^4,

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NO == RDO + p u (z^3 + z^4) z^2 + p NO z^2 + p u S (z^3 + z^4) z^2 + p SNO z^2,
R == p u (z^3 + z^4) z^2 + p NO z^2 + p u R (z^3 + z^4) z^2 + p RNO z^2}

CellPrint["Eliminate all variables except S,u,z"]
Eliminate[eqn, {DO, NO, R}]
Collect[%, S, Simplify]
CellPrint["F=following function in z,u,S"]
F = (S^3 z^4 + S^2 z^2 (-2 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6) + z (1 + z)
      (-1 - (-1 + u) z^4 + z^5 + z^6 - 2 (-1 + u) z^7 - 2 (-1 + u) z^8 + (-1 + u)^2 z^9 + (-1 + u)^2 z^10)) / ((-1 + z)
      (1 - z + 2 z^3 - 2 u z^5 + z^6 + (-1 + 2 u) z^7 - 2 (-1 + u) z^8 - 2 (-1 + u) z^9 + (-1 + u)^2 z^10 + (-1 + u)^2 z^11))

f = (F /. S -> s)
s - f
(* express over a common denominator*)
Together[s - f]
a = Numerator[%]

dfs = D[f, s]
1 - dfs
(* express over a common denominator*)
Together[1 - dfs]
(* a is numerator of s-f, and b is numerator of 1-dfs,
where both have identical denominators *)
b = Numerator[%]
(* a is numerator of s-f, and b is numerator of 1-dfs,
where both have identical denominators *)
If[Denominator[Together[s - f]] == Denominator[Together[1 - dfs]],
  CellPrint["Denominator of (s-f) same as that of (1-dfs)"],
  CellPrint["Denominator of (s-f) different than than of (1-dfs)"]]

(* NOTE: denominators of both expressions s-f and 1-dfs is the same *)
CellPrint["Now compute the resultant of numerators a,b to get relation between S,u,z"]

(* res =Resultant[s-f,1-dfs,s] *)

(*We compute resultant of numerators,
since we have Resultant[S-F,1-D[F,S],S]=0 which is not much use.*)
CellPrint["Express S-F and 1-D[F,S] as fractions over the same common denominator"]
CellPrint["Then compute the resultant of the numerators of these expressions"]
res = Resultant[a, b, s]
(* Replace z by z[u], a function of u *)

res /. z -> z[u]
(* Now compute z'[u] *)
dres = D[%, u]
Simplify[Collect[dres, z'[u]]]
Solve[dres == 0, z'[u]]
dzu = Last[Last[Last[Solve[dres == 0, z'[u]]]]];
(*z[1] equals rho, the dominant singularity *)

rho = 0.47550096719066426;
(* value of z0 in the first part of this file, the dominant singularity*)
dzuEvaluatedAt1 = (dzu /. u -> 1) /. z[1] -> rho
CellPrint[
  "According to Drmota's Theorem 1, the mean equals -z'[1]/z[1], computed next. "]

mu = ((-dzu / z[u] /. u -> 1) /. z[1] -> rho)

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(*Variance computation *)
(* d2zu is z'[1] *)
d2zu = ((D[dzu, u] /. u -> 1) /. z[1] -> rho) /. z'[1] -> dzuEvaluatedAt1
CellPrint["Now compute variance, which by DrmotTheta is -z'[1]/z[1] + mu^2 + mu"]
var = -d2zu / rho + mu * mu + mu
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$$\{S = D0 + N0, D0 = z + z^2 + z^3 + z^4, N0 = D0 R + N0 z^2 + N0 S z^2 + u z^2 (z^3 + z^4) + S u z^2 (z^3 + z^4), \\ R = N0 z^2 + N0 R z^2 + u z^2 (z^3 + z^4) + R u z^2 (z^3 + z^4)\}$$

Eliminate all variables except S,u,z

$$S^3 z^4 + S^2 z^2 (-2 + z^2 - 2 z^3 - 2 z^4 - 2 z^5 + 2 u z^5 - 2 z^6 + 2 u z^6) + \\ S (1 - z^2 + 2 z^3 + 2 z^4 - 2 u z^5 + z^6 - 2 u z^6 + 2 u z^7 + z^8 + 4 z^9 - 4 u z^9 + \\ 3 z^{10} - 4 u z^{10} + u^2 z^{10} + 2 z^{11} - 4 u z^{11} + 2 u^2 z^{11} + z^{12} - 2 u z^{12} + u^2 z^{12}) = \\ z (1 + z - z^4 + u z^4 - 2 z^5 + u z^5 - 2 z^6 - 3 z^7 + 2 u z^7 - 4 z^8 + 4 u z^8 - 3 z^9 + 4 u z^9 - \\ u^2 z^9 - 2 z^{10} + 4 u z^{10} - 2 u^2 z^{10} - z^{11} + 2 u z^{11} - u^2 z^{11}) \\ S^3 z^4 + S^2 z^2 (-2 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6) + S (1 + z) \\ (1 - z + 2 z^3 - 2 u z^5 + z^6 + (-1 + 2 u) z^7 - 2 (-1 + u) z^8 - 2 (-1 + u) z^9 + (-1 + u)^2 z^{10} + (-1 + u)^2 z^{11}) = \\ -z (1 + z) (-1 - (-1 + u) z^4 + z^5 + z^6 - 2 (-1 + u) z^7 - 2 (-1 + u) z^8 + (-1 + u)^2 z^9 + (-1 + u)^2 z^{10})$$

F=following function in z,u,S

$$(S^3 z^4 + S^2 z^2 (-2 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6) + z (1 + z) \\ (-1 - (-1 + u) z^4 + z^5 + z^6 - 2 (-1 + u) z^7 - 2 (-1 + u) z^8 + (-1 + u)^2 z^9 + (-1 + u)^2 z^{10})) / ((-1 - z) \\ (1 - z + 2 z^3 - 2 u z^5 + z^6 + (-1 + 2 u) z^7 - 2 (-1 + u) z^8 - 2 (-1 + u) z^9 + (-1 + u)^2 z^{10} + (-1 + u)^2 z^{11})) \\ (S^3 z^4 + S^2 z^2 (-2 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6) + z (1 + z) \\ (-1 - (-1 + u) z^4 + z^5 + z^6 - 2 (-1 + u) z^7 - 2 (-1 + u) z^8 + (-1 + u)^2 z^9 + (-1 + u)^2 z^{10})) / ((-1 - z) \\ (1 - z + 2 z^3 - 2 u z^5 + z^6 + (-1 + 2 u) z^7 - 2 (-1 + u) z^8 - 2 (-1 + u) z^9 + (-1 + u)^2 z^{10} + (-1 + u)^2 z^{11})) \\ s - (S^3 z^4 + S^2 z^2 (-2 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6) + z (1 + z) \\ (-1 - (-1 + u) z^4 + z^5 + z^6 - 2 (-1 + u) z^7 - 2 (-1 + u) z^8 + (-1 + u)^2 z^9 + (-1 + u)^2 z^{10})) / ((-1 - z) \\ (1 - z + 2 z^3 - 2 u z^5 + z^6 + (-1 + 2 u) z^7 - 2 (-1 + u) z^8 - 2 (-1 + u) z^9 + (-1 + u)^2 z^{10} + (-1 + u)^2 z^{11})) \\ (s - z - z^2 - s z^2 - 2 s^2 z^2 + 2 s z^3 + 2 s z^4 + s^2 z^4 + s^3 z^4 + z^5 - 2 s^2 z^5 - u z^5 - 2 s u z^5 + 2 z^6 + s z^6 - 2 s^2 z^6 - \\ u z^6 - 2 s u z^6 + 2 z^7 - 2 s^2 z^7 + 2 s u z^7 + 2 s^2 u z^7 + 3 z^8 + s z^8 - 2 s^2 z^8 - 2 u z^8 + 2 s^2 u z^8 + 4 z^9 + \\ 4 s z^9 - 4 u z^9 - 4 s u z^9 + 3 z^{10} + 3 s z^{10} - 4 u z^{10} - 4 s u z^{10} + u^2 z^{10} + s u^2 z^{10} + 2 z^{11} + 2 s z^{11} - \\ 4 u z^{11} - 4 s u z^{11} + 2 u^2 z^{11} + 2 s u^2 z^{11} + z^{12} + s z^{12} - 2 u z^{12} - 2 s u z^{12} + u^2 z^{12} + s u^2 z^{12}) / \\ ((1 + z) (1 - z + 2 z^3 - 2 u z^5 + z^6 - z^7 + 2 u z^7 + 2 z^8 - 2 u z^8 + 2 z^9 - \\ 2 u z^9 + z^{10} - 2 u z^{10} + u^2 z^{10} + z^{11} - 2 u z^{11} + u^2 z^{11})) \\ s - z - z^2 - s z^2 - 2 s^2 z^2 + 2 s z^3 + 2 s z^4 + s^2 z^4 + s^3 z^4 + z^5 - 2 s^2 z^5 - u z^5 - 2 s u z^5 + 2 z^6 + s z^6 - \\ 2 s^2 z^6 - u z^6 - 2 s u z^6 + 2 z^7 - 2 s^2 z^7 + 2 s u z^7 + 2 s^2 u z^7 + 3 z^8 + s z^8 - 2 s^2 z^8 - 2 u z^8 + 2 s^2 u z^8 + \\ 4 z^9 + 4 s z^9 - 4 u z^9 - 4 s u z^9 + 3 z^{10} + 3 s z^{10} - 4 u z^{10} - 4 s u z^{10} + u^2 z^{10} + s u^2 z^{10} + 2 z^{11} + \\ 2 s z^{11} - 4 u z^{11} - 4 s u z^{11} + 2 u^2 z^{11} + 2 s u^2 z^{11} + z^{12} + s z^{12} - 2 u z^{12} - 2 s u z^{12} + u^2 z^{12} + s u^2 z^{12} \\ (3 s^2 z^4 + 2 s z^2 (-2 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6)) / ((-1 - z) \\ (1 - z + 2 z^3 - 2 u z^5 + z^6 + (-1 + 2 u) z^7 - 2 (-1 + u) z^8 - 2 (-1 + u) z^9 + (-1 + u)^2 z^{10} + (-1 + u)^2 z^{11})) \\ 1 - (3 s^2 z^4 + 2 s z^2 (-2 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6)) / ((-1 - z) \\ (1 - z + 2 z^3 - 2 u z^5 + z^6 + (-1 + 2 u) z^7 - 2 (-1 + u) z^8 - 2 (-1 + u) z^9 + (-1 + u)^2 z^{10} + (-1 + u)^2 z^{11}))$$

$$\begin{aligned}
& (1 - z^2 - 4 s z^2 + 2 z^3 + 2 z^4 + 2 s z^4 + 3 s^2 z^4 - 4 s z^5 - \\
& \quad 2 u z^5 + z^6 - 4 s z^6 - 2 u z^6 - 4 s z^7 + 2 u z^7 + 4 s u z^7 + z^8 - 4 s z^8 + 4 s u z^8 + 4 z^9 - \\
& \quad 4 u z^9 + 3 z^{10} - 4 u z^{10} + u^2 z^{10} + 2 z^{11} - 4 u z^{11} + 2 u^2 z^{11} + z^{12} - 2 u z^{12} + u^2 z^{12}) / \\
& \left( (1 + z) (1 - z + 2 z^3 - 2 u z^5 + z^6 - z^7 + 2 u z^7 + 2 z^8 - 2 u z^8 + 2 z^9 - \right. \\
& \quad \left. 2 u z^9 + z^{10} - 2 u z^{10} + u^2 z^{10} + z^{11} - 2 u z^{11} + u^2 z^{11}) \right) \\
& 1 - z^2 - 4 s z^2 + 2 z^3 + 2 z^4 + 2 s z^4 + 3 s^2 z^4 - 4 s z^5 - 2 u z^5 + z^6 - 4 s z^6 - 2 u z^6 - 4 s z^7 + 2 u z^7 + 4 s u z^7 + \\
& \quad z^8 - 4 s z^8 + 4 s u z^8 + 4 z^9 - 4 u z^9 + 3 z^{10} - 4 u z^{10} + u^2 z^{10} + 2 z^{11} - 4 u z^{11} + 2 u^2 z^{11} + z^{12} - 2 u z^{12} + u^2 z^{12}
\end{aligned}$$

Denominator of (s-f) same as that of (1-dfs)

Now compute the resultant of numerators a,b to get relation between S,u,z

Express S-F and 1-D[F,S] as fractions over the same common denominator

Then compute the resultant of the numerators of these expressions

$$\begin{aligned}
& -4 z^{11} - 5 z^{12} + 2 z^{13} + 19 z^{14} + 40 z^{15} + 48 z^{16} + 12 u z^{16} + 34 z^{17} + 26 u z^{17} - 24 z^{18} + 32 u z^{18} - 110 z^{19} + \\
& \quad 62 u z^{19} - 208 z^{20} + 112 u z^{20} - 296 z^{21} + 184 u z^{21} - 12 u^2 z^{21} - 349 z^{22} + 286 u z^{22} - 37 u^2 z^{22} - \\
& \quad 370 z^{23} + 364 u z^{23} - 62 u^2 z^{23} - 353 z^{24} + 410 u z^{24} - 97 u^2 z^{24} - 316 z^{25} + 432 u z^{25} - 132 u^2 z^{25} - \\
& \quad 260 z^{26} + 408 u z^{26} - 156 u^2 z^{26} + 4 u^3 z^{26} - 196 z^{27} + 360 u z^{27} - 180 u^2 z^{27} + 16 u^3 z^{27} - 136 z^{28} + \\
& \quad 288 u z^{28} - 180 u^2 z^{28} + 28 u^3 z^{28} - 80 z^{29} + 192 u z^{29} - 144 u^2 z^{29} + 32 u^3 z^{29} - 40 z^{30} + 108 u z^{30} - \\
& \quad 96 u^2 z^{30} + 28 u^3 z^{30} - 16 z^{31} + 48 u z^{31} - 48 u^2 z^{31} + 16 u^3 z^{31} - 4 z^{32} + 12 u z^{32} - 12 u^2 z^{32} + 4 u^3 z^{32} \\
& -4 z[u]^{11} - 5 z[u]^{12} + 2 z[u]^{13} + 19 z[u]^{14} + 40 z[u]^{15} + 48 z[u]^{16} + 12 u z[u]^{16} + 34 z[u]^{17} + 26 u z[u]^{17} - \\
& \quad 24 z[u]^{18} + 32 u z[u]^{18} - 110 z[u]^{19} + 62 u z[u]^{19} - 208 z[u]^{20} + 112 u z[u]^{20} - 296 z[u]^{21} + \\
& \quad 184 u z[u]^{21} - 12 u^2 z[u]^{21} - 349 z[u]^{22} + 286 u z[u]^{22} - 37 u^2 z[u]^{22} - 370 z[u]^{23} + 364 u z[u]^{23} - \\
& \quad 62 u^2 z[u]^{23} - 353 z[u]^{24} + 410 u z[u]^{24} - 97 u^2 z[u]^{24} - 316 z[u]^{25} + 432 u z[u]^{25} - 132 u^2 z[u]^{25} - \\
& \quad 260 z[u]^{26} + 408 u z[u]^{26} - 156 u^2 z[u]^{26} + 4 u^3 z[u]^{26} - 196 z[u]^{27} + 360 u z[u]^{27} - 180 u^2 z[u]^{27} + \\
& \quad 16 u^3 z[u]^{27} - 136 z[u]^{28} + 288 u z[u]^{28} - 180 u^2 z[u]^{28} + 28 u^3 z[u]^{28} - 80 z[u]^{29} + 192 u z[u]^{29} - \\
& \quad 144 u^2 z[u]^{29} + 32 u^3 z[u]^{29} - 40 z[u]^{30} + 108 u z[u]^{30} - 96 u^2 z[u]^{30} + 28 u^3 z[u]^{30} - 16 z[u]^{31} + \\
& \quad 48 u z[u]^{31} - 48 u^2 z[u]^{31} + 16 u^3 z[u]^{31} - 4 z[u]^{32} + 12 u z[u]^{32} - 12 u^2 z[u]^{32} + 4 u^3 z[u]^{32}
\end{aligned}$$

$$\begin{aligned}
& 12 z[u]^{16} + 26 z[u]^{17} + 32 z[u]^{18} + 62 z[u]^{19} + 112 z[u]^{20} + 184 z[u]^{21} - 24 u z[u]^{21} + \\
& 286 z[u]^{22} - 74 u z[u]^{22} + 364 z[u]^{23} - 124 u z[u]^{23} + 410 z[u]^{24} - 194 u z[u]^{24} + 432 z[u]^{25} - \\
& 264 u z[u]^{25} + 408 z[u]^{26} - 312 u z[u]^{26} + 12 u^2 z[u]^{26} + 360 z[u]^{27} - 360 u z[u]^{27} + \\
& 48 u^2 z[u]^{27} + 288 z[u]^{28} - 360 u z[u]^{28} + 84 u^2 z[u]^{28} + 192 z[u]^{29} - 288 u z[u]^{29} + \\
& 96 u^2 z[u]^{29} + 108 z[u]^{30} - 192 u z[u]^{30} + 84 u^2 z[u]^{30} + 48 z[u]^{31} - 96 u z[u]^{31} + \\
& 48 u^2 z[u]^{31} + 12 z[u]^{32} - 24 u z[u]^{32} + 12 u^2 z[u]^{32} - 44 z[u]^{10} \text{Derivative}[1][z][u] - \\
& 60 z[u]^{11} \text{Derivative}[1][z][u] + 26 z[u]^{12} \text{Derivative}[1][z][u] + 266 z[u]^{13} \text{Derivative}[1][z][u] + \\
& 600 z[u]^{14} \text{Derivative}[1][z][u] + 768 z[u]^{15} \text{Derivative}[1][z][u] + \\
& 192 u z[u]^{15} \text{Derivative}[1][z][u] + 578 z[u]^{16} \text{Derivative}[1][z][u] + \\
& 442 u z[u]^{16} \text{Derivative}[1][z][u] - 432 z[u]^{17} \text{Derivative}[1][z][u] + \\
& 576 u z[u]^{17} \text{Derivative}[1][z][u] - 2090 z[u]^{18} \text{Derivative}[1][z][u] + \\
& 1178 u z[u]^{18} \text{Derivative}[1][z][u] - 4160 z[u]^{19} \text{Derivative}[1][z][u] + \\
& 2240 u z[u]^{19} \text{Derivative}[1][z][u] - 6216 z[u]^{20} \text{Derivative}[1][z][u] + \\
& 3864 u z[u]^{20} \text{Derivative}[1][z][u] - 252 u^2 z[u]^{20} \text{Derivative}[1][z][u] - \\
& 7678 z[u]^{21} \text{Derivative}[1][z][u] + 6292 u z[u]^{21} \text{Derivative}[1][z][u] - \\
& 814 u^2 z[u]^{21} \text{Derivative}[1][z][u] - 8510 z[u]^{22} \text{Derivative}[1][z][u] + \\
& 8372 u z[u]^{22} \text{Derivative}[1][z][u] - 1426 u^2 z[u]^{22} \text{Derivative}[1][z][u] - \\
& 8472 z[u]^{23} \text{Derivative}[1][z][u] + 9840 u z[u]^{23} \text{Derivative}[1][z][u] - \\
& 2328 u^2 z[u]^{23} \text{Derivative}[1][z][u] - 7900 z[u]^{24} \text{Derivative}[1][z][u] + \\
& 10800 u z[u]^{24} \text{Derivative}[1][z][u] - 3300 u^2 z[u]^{24} \text{Derivative}[1][z][u] - \\
& 6760 z[u]^{25} \text{Derivative}[1][z][u] + 10608 u z[u]^{25} \text{Derivative}[1][z][u] - \\
& 4056 u^2 z[u]^{25} \text{Derivative}[1][z][u] + 104 u^3 z[u]^{25} \text{Derivative}[1][z][u] - \\
& 5292 z[u]^{26} \text{Derivative}[1][z][u] + 9720 u z[u]^{26} \text{Derivative}[1][z][u] - \\
& 4860 u^2 z[u]^{26} \text{Derivative}[1][z][u] + 432 u^3 z[u]^{26} \text{Derivative}[1][z][u] - \\
& 3808 z[u]^{27} \text{Derivative}[1][z][u] + 8064 u z[u]^{27} \text{Derivative}[1][z][u] - \\
& 5040 u^2 z[u]^{27} \text{Derivative}[1][z][u] + 784 u^3 z[u]^{27} \text{Derivative}[1][z][u] - \\
& 2320 z[u]^{28} \text{Derivative}[1][z][u] + 5568 u z[u]^{28} \text{Derivative}[1][z][u] - \\
& 4176 u^2 z[u]^{28} \text{Derivative}[1][z][u] + 928 u^3 z[u]^{28} \text{Derivative}[1][z][u] - \\
& 1200 z[u]^{29} \text{Derivative}[1][z][u] + 3240 u z[u]^{29} \text{Derivative}[1][z][u] - \\
& 2880 u^2 z[u]^{29} \text{Derivative}[1][z][u] + 840 u^3 z[u]^{29} \text{Derivative}[1][z][u] - \\
& 496 z[u]^{30} \text{Derivative}[1][z][u] + 1488 u z[u]^{30} \text{Derivative}[1][z][u] - \\
& 1488 u^2 z[u]^{30} \text{Derivative}[1][z][u] + 496 u^3 z[u]^{30} \text{Derivative}[1][z][u] - \\
& 128 z[u]^{31} \text{Derivative}[1][z][u] + 384 u z[u]^{31} \text{Derivative}[1][z][u] - \\
& 384 u^2 z[u]^{31} \text{Derivative}[1][z][u] + 128 u^3 z[u]^{31} \text{Derivative}[1][z][u]
\end{aligned}$$

$$\begin{aligned}
& 2 z[u]^{10} (6 (-1 + u)^2 z[u]^{22} - 22 \text{Derivative}[1][z][u] - \\
& 30 z[u] \text{Derivative}[1][z][u] + 13 z[u]^2 \text{Derivative}[1][z][u] + \\
& 133 z[u]^3 \text{Derivative}[1][z][u] + 300 z[u]^4 \text{Derivative}[1][z][u] + \\
& 96 (4 + u) z[u]^5 \text{Derivative}[1][z][u] + 8 (-1 + u)^2 z[u]^{21} (3 + 8 (-1 + u) \text{Derivative}[1][z][u]) + \\
& 2 (-1 + u) z[u]^{20} (3 (-9 + 7 u) + 124 (-1 + u)^2 \text{Derivative}[1][z][u]) + \\
& 2 z[u]^{18} (72 - 90 u + 21 u^2 + 116 (-1 + u)^2 (-5 + 2 u) \text{Derivative}[1][z][u]) + \\
& z[u]^7 (13 + 72 (-3 + 4 u) \text{Derivative}[1][z][u]) + \\
& z[u]^9 (31 + 160 (-13 + 7 u) \text{Derivative}[1][z][u]) + \\
& z[u]^6 (6 + 17 (17 + 13 u) \text{Derivative}[1][z][u]) + \\
& z[u]^8 (16 + 19 (-55 + 31 u) \text{Derivative}[1][z][u]) - \\
& 14 z[u]^{10} (-4 + 3 (74 - 46 u + 3 u^2) \text{Derivative}[1][z][u]) + \\
& 12 (-1 + u) z[u]^{19} (4 (-2 + u) + 5 (10 - 17 u + 7 u^2) \text{Derivative}[1][z][u]) + \\
& z[u]^{12} (143 - 37 u - 23 (185 - 182 u + 31 u^2) \text{Derivative}[1][z][u]) + \\
& z[u]^{14} (205 - 97 u - 50 (79 - 108 u + 33 u^2) \text{Derivative}[1][z][u]) + \\
& z[u]^{11} (92 - 12 u - 11 (349 - 286 u + 37 u^2) \text{Derivative}[1][z][u]) - \\
& 2 z[u]^{13} (-91 + 31 u + 6 (353 - 410 u + 97 u^2) \text{Derivative}[1][z][u]) + \\
& 4 z[u]^{15} (54 - 33 u + 13 (-65 + 102 u - 39 u^2 + u^3) \text{Derivative}[1][z][u]) + \\
& 6 z[u]^{16} (34 - 26 u + u^2 + 9 (-49 + 90 u - 45 u^2 + 4 u^3) \text{Derivative}[1][z][u]) + \\
& 4 z[u]^{17} (45 - 45 u + 6 u^2 + 14 (-34 + 72 u - 45 u^2 + 7 u^3) \text{Derivative}[1][z][u])) \\
& \{ \{ \text{Derivative}[1][z][u] \rightarrow \\
& (-6 z[u]^6 - 13 z[u]^7 - 16 z[u]^8 - 31 z[u]^9 - 56 z[u]^{10} - 92 z[u]^{11} + 12 u z[u]^{11} - 143 z[u]^{12} + \\
& 37 u z[u]^{12} - 182 z[u]^{13} + 62 u z[u]^{13} - 205 z[u]^{14} + 97 u z[u]^{14} - 216 z[u]^{15} + 132 u z[u]^{15} - \\
& 204 z[u]^{16} + 156 u z[u]^{16} - 6 u^2 z[u]^{16} - 180 z[u]^{17} + 180 u z[u]^{17} - 24 u^2 z[u]^{17} - 144 z[u]^{18} + \\
& 180 u z[u]^{18} - 42 u^2 z[u]^{18} - 96 z[u]^{19} + 144 u z[u]^{19} - 48 u^2 z[u]^{19} - 54 z[u]^{20} + 96 u z[u]^{20} - \\
& 42 u^2 z[u]^{20} - 24 z[u]^{21} + 48 u z[u]^{21} - 24 u^2 z[u]^{21} - 6 z[u]^{22} + 12 u z[u]^{22} - 6 u^2 z[u]^{22}) / \\
& (-22 - 30 z[u] + 13 z[u]^2 + 133 z[u]^3 + 300 z[u]^4 + 384 z[u]^5 + 96 u z[u]^5 + 289 z[u]^6 + 221 u z[u]^6 - \\
& 216 z[u]^7 + 288 u z[u]^7 - 1045 z[u]^8 + 589 u z[u]^8 - 2080 z[u]^9 + 1120 u z[u]^9 - 3108 z[u]^{10} + \\
& 1932 u z[u]^{10} - 126 u^2 z[u]^{10} - 3839 z[u]^{11} + 3146 u z[u]^{11} - 407 u^2 z[u]^{11} - 4255 z[u]^{12} + \\
& 4186 u z[u]^{12} - 713 u^2 z[u]^{12} - 4236 z[u]^{13} + 4920 u z[u]^{13} - 1164 u^2 z[u]^{13} - 3950 z[u]^{14} + \\
& 5400 u z[u]^{14} - 1650 u^2 z[u]^{14} - 3380 z[u]^{15} + 5304 u z[u]^{15} - 2028 u^2 z[u]^{15} + 52 u^3 z[u]^{15} - \\
& 2646 z[u]^{16} + 4860 u z[u]^{16} - 2430 u^2 z[u]^{16} + 216 u^3 z[u]^{16} - 1904 z[u]^{17} + 4032 u z[u]^{17} - \\
& 2520 u^2 z[u]^{17} + 392 u^3 z[u]^{17} - 1160 z[u]^{18} + 2784 u z[u]^{18} - 2088 u^2 z[u]^{18} + 464 u^3 z[u]^{18} - \\
& 600 z[u]^{19} + 1620 u z[u]^{19} - 1440 u^2 z[u]^{19} + 420 u^3 z[u]^{19} - 248 z[u]^{20} + 744 u z[u]^{20} - \\
& 744 u^2 z[u]^{20} + 248 u^3 z[u]^{20} - 64 z[u]^{21} + 192 u z[u]^{21} - 192 u^2 z[u]^{21} + 64 u^3 z[u]^{21}) \} \} \\
& - 0.0284537
\end{aligned}$$

According to Drmota's Theorem 1, the mean equals  $-z'[1]/z[1]$ , computed next.

0.0598393

0.0215904

Now compute variance, which by Drmota is  $-z''[1]/z[1] + \mu^2 + \mu$

0.0180144