# (∗ Asymptotic expected number of hairpins in saturated structures with theta= 3 and stickiness p=1/2∗)

**(∗We first compute the dominant singularity and asymptotic number of saturated structures, using Drmota−Lalley−Woods Theorem. ∗)**

```
Clear["*"]
Clear[p, S, D0, N0, z, R, eqn0, eqn, F, z0, y0, dFdzOfz0S0, d2FdyOfz0S0];
p = 1 / 2;
eqn = {S == D0 + N0, D0 == z + z^2 + z^3 + z^4,
    N0 == R D0 +  p (z^3 + z^4) z^2 + p N0 z^2 + p S (z^3 + z^4) z^2 + p S N0 z^2,
    R == p (z^3 + z^4) z^2 + p N0 z^2 + p R (z^3 + z^4) z^2 + p R N0 z^2};
Eliminate[eqn, {N0, D0, R}]
F = (S^3 z^4 + S^2 z^2 (-4 + z^2 - 2 z^3 - 2 z^4) - z (4 + 4 z + 2 z^2 + 2 z^3 - z^5 - 2 z^6 - z^7)) /
    (- (4 - 2 z^2 + 4 z^3 + 4 z^4 - 2 z^5 - z^6 + 2 z^7 + z^8))
```

```
NSolve[{F == S, D[F, S] == 1}, {z, S}];
z0 = 0.5729259800074672`
y0 = 2.0778447462496445`

dFdzOfz0S0 = D[F, z] /. {z → z0, S → y0}

d2FdyOfz0S0 = D[F, {S, 2}] /. {z → z0, S → y0}
c = Sqrt[z0 dFdzOfz0S0 / (2 Pi d2FdyOfz0S0)]
c * (1 / z0) ^n n^ (-3 / 2)
```

$$S^3 z^4 + S^2 z^2 \left(-4 + z^2 - 2 z^3 - 2 z^4\right) + S \left(4 - 2 z^2 + 4 z^3 + 4 z^4 - 2 z^5 - z^6 + 2 z^7 + z^8\right) ==$$
$$z \left(4 + 4 z + 2 z^2 + 2 z^3 - z^5 - 2 z^6 - z^7\right)$$

$$\left(S^3 z^4 + S^2 z^2 \left(-4 + z^2 - 2 z^3 - 2 z^4\right) - z \left(4 + 4 z + 2 z^2 + 2 z^3 - z^5 - 2 z^6 - z^7\right)\right) /$$
$$\left(-4 + 2 z^2 - 4 z^3 - 4 z^4 + 2 z^5 + z^6 - 2 z^7 - z^8\right)$$

0.572926

2.07784

4.66968

0.329317

1.13709

$$\frac{1.13709 \times 1.74543^n}{n^{3/2}}$$

**(∗Now, we compute mean,variance using Drmota's Theorem∗)**

```
Clear["*"]
p = 1 / 2;
eqn = {S == D0 + N0, D0 == z + z^2 + z^3 + z^4,
    N0 == R D0 +  p u (z^3 + z^4) z^2 + p N0 z^2 + p u S (z^3 + z^4) z^2 + p S N0 z^2,
    R == p u (z^3 + z^4) z^2 + p N0 z^2 + p u R (z^3 + z^4) z^2 + p R N0 z^2};

CellPrint["Eliminate all variables except S,u,z"]
Eliminate[eqn, {D0, N0, R}]
Collect[%, S, Simplify]
```

$$F = \left(S^3 z^4 + S^2 z^2 \left(-4 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6\right) + z (1 + z)\right.$$
$$\left(-4 - 2 z^2 - 2 (-1 + u) z^4 + z^5 + z^6 - 2 (-1 + u) z^7 - 2 (-1 + u) z^8 + (-1 + u)^2 z^9 + (-1 + u)^2 z^{10}\right)\right) /$$
$$\left(- \left(4 - 2 z^2 + 4 z^3 + 4 z^4 + (2 - 4 u) z^5 + (3 - 4 u) z^6 + 2 u z^7 + z^8 - 4 (-1 + u) z^9 + \right.\right.$$
$$\left.\left. \left(3 - 4 u + u^2\right) z^{10} + 2 (-1 + u)^2 z^{11} + (-1 + u)^2 z^{12}\right)\right)$$

```
f = (F /. S → s)
s - f
(* express over a common denominator*)
Together[s - f]
a = Numerator[%]

dfs = D[f, s]
1 - dfs
(* express over a common denominator*)
Together[1 - dfs]
(* a is numerator of s-f, and b is numerator of 1-dfs,
where both have identical denominators *)
b = Numerator[%]
(* a is numerator of s-f, and b is numerator of 1-dfs,
where both have identical denominators *)
If[Denominator[Together[s - f]] == Denominator[Together[1 - dfs]],
 CellPrint["Denominator of (s-f) same as that of (1-dfs)"],
 CellPrint["Denominator of (s-f) different than than of (1-dfs)"]]
CellPrint["Now compute the resultant of numerators a,b to get relation between S,u,z"]

(* res =Resultant[s-f,1-dfs,s] *)


(*We compute resultant of numerators,
since we have Resultant[S-F,1-D[F,S],S]=0 which is not much use.*)
CellPrint["Express S-F and 1-D[F,S] as fractions over the same common denominator"]
CellPrint["Then compute the resultant of the numerators of these expressions"]
res = Resultant[a, b, s]
(* Replace z by z[u], a function of u *)


res /. z → z[u]
(* Now compute z'[u] *)
dres = D[%, u]
Simplify[Collect[dres, z '[u]]]
Solve[dres == 0, z '[u]]
dzu = Last[Last[Last[Solve[dres == 0, z '[u]]]]];
(*z[1] equals rho, the dominant singularity *)




rho = 0.5729259800074672`;
(* value of z0 in the first part of this file, the dominant singularity*)
dzuEvaluatedAt1 = (dzu /. u → 1) /. z[1] → rho
CellPrint[
```

```
   "According to Drmota's Theorem 1, the mean equals -z'[1]/z[1], computed next. "]

mu = ((-dzu / z[u] /. u → 1) /. z[1] → rho)

(*Variance computation *)
(* d2zu is z''[1] *)
d2zu = (((D[dzu, u] /. u → 1) /. z[1] → rho) /. z'[1] → dzuEvaluatedAt1)
CellPrint["Now compute variance, which by Drmota is -z''[1]/z[1] + mu^2 + mu"]
var = -d2zu / rho + mu * mu + mu
```

Eliminate all variables except S,u,z

$$S^3 z^4 + S^2 z^2 \left(-4 + z^2 - 2 z^3 - 2 z^4 - 2 z^5 + 2 u z^5 - 2 z^6 + 2 u z^6\right) +$$
$$S \left(4 - 2 z^2 + 4 z^3 + 4 z^4 + 2 z^5 - 4 u z^5 + 3 z^6 - 4 u z^6 + 2 u z^7 + z^8 + 4 z^9 - \right.$$
$$\left. 4 u z^9 + 3 z^{10} - 4 u z^{10} + u^2 z^{10} + 2 z^{11} - 4 u z^{11} + 2 u^2 z^{11} + z^{12} - 2 u z^{12} + u^2 z^{12}\right) ==$$
$$z \left(4 + 4 z + 2 z^2 + 2 z^3 - 2 z^4 + 2 u z^4 - 3 z^5 + 2 u z^5 - 2 z^6 - 3 z^7 + 2 u z^7 - 4 z^8 + 4 u z^8 - \right.$$
$$\left. 3 z^9 + 4 u z^9 - u^2 z^9 - 2 z^{10} + 4 u z^{10} - 2 u^2 z^{10} - z^{11} + 2 u z^{11} - u^2 z^{11}\right)$$

$$S^3 z^4 + S^2 z^2 \left(-4 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6\right) +$$
$$S \left(4 - 2 z^2 + 4 z^3 + 4 z^4 + (2 - 4 u) z^5 + (3 - 4 u) z^6 + 2 u z^7 + z^8 - \right.$$
$$\left. 4 (-1 + u) z^9 + (3 - 4 u + u^2) z^{10} + 2 (-1 + u)^2 z^{11} + (-1 + u)^2 z^{12}\right) ==$$
$$-z (1 + z) \left(-4 - 2 z^2 - 2 (-1 + u) z^4 + z^5 + z^6 - 2 (-1 + u) z^7 - 2 (-1 + u) z^8 + (-1 + u)^2 z^9 + (-1 + u)^2 z^{10}\right)$$

$$\left(S^3 z^4 + S^2 z^2 \left(-4 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6\right) + z (1 + z) \right.$$
$$\left. \left(-4 - 2 z^2 - 2 (-1 + u) z^4 + z^5 + z^6 - 2 (-1 + u) z^7 - 2 (-1 + u) z^8 + (-1 + u)^2 z^9 + (-1 + u)^2 z^{10}\right)\right) /$$
$$\left(-4 + 2 z^2 - 4 z^3 - 4 z^4 - (2 - 4 u) z^5 - (3 - 4 u) z^6 - 2 u z^7 - z^8 + 4 (-1 + u) z^9 - \right.$$
$$\left. (3 - 4 u + u^2) z^{10} - 2 (-1 + u)^2 z^{11} - (-1 + u)^2 z^{12}\right)$$

$$\left(s^3 z^4 + s^2 z^2 \left(-4 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6\right) + z (1 + z) \right.$$
$$\left. \left(-4 - 2 z^2 - 2 (-1 + u) z^4 + z^5 + z^6 - 2 (-1 + u) z^7 - 2 (-1 + u) z^8 + (-1 + u)^2 z^9 + (-1 + u)^2 z^{10}\right)\right) /$$
$$\left(-4 + 2 z^2 - 4 z^3 - 4 z^4 - (2 - 4 u) z^5 - (3 - 4 u) z^6 - 2 u z^7 - z^8 + 4 (-1 + u) z^9 - \right.$$
$$\left. (3 - 4 u + u^2) z^{10} - 2 (-1 + u)^2 z^{11} - (-1 + u)^2 z^{12}\right)$$

$$s - \left(s^3 z^4 + s^2 z^2 \left(-4 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6\right) + z (1 + z) \right.$$
$$\left. \left(-4 - 2 z^2 - 2 (-1 + u) z^4 + z^5 + z^6 - 2 (-1 + u) z^7 - 2 (-1 + u) z^8 + (-1 + u)^2 z^9 + (-1 + u)^2 z^{10}\right)\right) /$$
$$\left(-4 + 2 z^2 - 4 z^3 - 4 z^4 - (2 - 4 u) z^5 - (3 - 4 u) z^6 - 2 u z^7 - z^8 + 4 (-1 + u) z^9 - \right.$$
$$\left. (3 - 4 u + u^2) z^{10} - 2 (-1 + u)^2 z^{11} - (-1 + u)^2 z^{12}\right)$$

$$\left(4 s - 4 z - 4 z^2 - 2 s z^2 - 4 s^2 z^2 - 2 z^3 + 4 s z^3 - 2 z^4 + 4 s z^4 + s^2 z^4 + s^3 z^4 + \right.$$
$$2 z^5 + 2 s z^5 - 2 s^2 z^5 - 2 u z^5 - 4 s u z^5 + 3 z^6 + 3 s z^6 - 2 s^2 z^6 - 2 u z^6 - 4 s u z^6 + 2 z^7 -$$
$$2 s^2 z^7 + 2 s u z^7 + 2 s^2 u z^7 + 3 z^8 + s z^8 - 2 s^2 z^8 - 2 u z^8 + 2 s^2 u z^8 + 4 z^9 + 4 s z^9 -$$
$$4 u z^9 - 4 s u z^9 + 3 z^{10} + 3 s z^{10} - 4 u z^{10} - 4 s u z^{10} + u^2 z^{10} + s u^2 z^{10} + 2 z^{11} + 2 s z^{11} -$$
$$4 u z^{11} - 4 s u z^{11} + 2 u^2 z^{11} + 2 s u^2 z^{11} + z^{12} + s z^{12} - 2 u z^{12} - 2 s u z^{12} + u^2 z^{12} + s u^2 z^{12}\right) /$$
$$\left(4 - 2 z^2 + 4 z^3 + 4 z^4 + 2 z^5 - 4 u z^5 + 3 z^6 - 4 u z^6 + 2 u z^7 + z^8 + 4 z^9 - 4 u z^9 + \right.$$
$$\left. 3 z^{10} - 4 u z^{10} + u^2 z^{10} + 2 z^{11} - 4 u z^{11} + 2 u^2 z^{11} + z^{12} - 2 u z^{12} + u^2 z^{12}\right)$$

$$4 s - 4 z - 4 z^2 - 2 s z^2 - 4 s^2 z^2 - 2 z^3 + 4 s z^3 - 2 z^4 + 4 s z^4 + s^2 z^4 + s^3 z^4 + 2 z^5 + 2 s z^5 - 2 s^2 z^5 - 2 u z^5 -$$
$$4 s u z^5 + 3 z^6 + 3 s z^6 - 2 s^2 z^6 - 2 u z^6 - 4 s u z^6 + 2 z^7 - 2 s^2 z^7 + 2 s u z^7 + 2 s^2 u z^7 + 3 z^8 + s z^8 - 2 s^2 z^8 -$$
$$2 u z^8 + 2 s^2 u z^8 + 4 z^9 + 4 s z^9 - 4 u z^9 - 4 s u z^9 + 3 z^{10} + 3 s z^{10} - 4 u z^{10} - 4 s u z^{10} + u^2 z^{10} + s u^2 z^{10} +$$
$$2 z^{11} + 2 s z^{11} - 4 u z^{11} - 4 s u z^{11} + 2 u^2 z^{11} + 2 s u^2 z^{11} + z^{12} + s z^{12} - 2 u z^{12} - 2 s u z^{12} + u^2 z^{12} + s u^2 z^{12}$$

$$\left(3 s^2 z^4 + 2 s z^2 \left(-4 + z^2 - 2 z^3 - 2 z^4 + 2 (-1 + u) z^5 + 2 (-1 + u) z^6\right)\right) /$$
$$\left(-4 + 2 z^2 - 4 z^3 - 4 z^4 - (2 - 4 u) z^5 - (3 - 4 u) z^6 - 2 u z^7 - \right.$$
$$\left. z^8 + 4 (-1 + u) z^9 - (3 - 4 u + u^2) z^{10} - 2 (-1 + u)^2 z^{11} - (-1 + u)^2 z^{12}\right)$$

$$1 - \left(3\, s^2\, z^4 + 2\, s\, z^2 \left(-4 + z^2 - 2\, z^3 - 2\, z^4 + 2\,(-1+u)\, z^5 + 2\,(-1+u)\, z^6\right)\right) \Big/$$
$$\left(-4 + 2\, z^2 - 4\, z^3 - 4\, z^4 - (2-4\,u)\, z^5 - (3-4\,u)\, z^6 - 2\,u\,z^7 - \right.$$
$$\left. z^8 + 4\,(-1+u)\, z^9 - \left(3 - 4\,u + u^2\right) z^{10} - 2\,(-1+u)^2\, z^{11} - (-1+u)^2\, z^{12}\right)$$

$$\left(4 - 2\, z^2 - 8\, s\, z^2 + 4\, z^3 + 4\, z^4 + 2\, s\, z^4 + 3\, s^2\, z^4 + 2\, z^5 - 4\, s\, z^5 - \right.$$
$$4\, u\, z^5 + 3\, z^6 - 4\, s\, z^6 - 4\, u\, z^6 - 4\, s\, z^7 + 2\, u\, z^7 + 4\, s\, u\, z^7 + z^8 - 4\, s\, z^8 + 4\, s\, u\, z^8 + 4\, z^9 -$$
$$\left. 4\, u\, z^9 + 3\, z^{10} - 4\, u\, z^{10} + u^2\, z^{10} + 2\, z^{11} - 4\, u\, z^{11} + 2\, u^2\, z^{11} + z^{12} - 2\, u\, z^{12} + u^2\, z^{12}\right) \Big/$$
$$\left(4 - 2\, z^2 + 4\, z^3 + 4\, z^4 + 2\, z^5 - 4\, u\, z^5 + 3\, z^6 - 4\, u\, z^6 + 2\, u\, z^7 + z^8 + 4\, z^9 - 4\, u\, z^9 + \right.$$
$$\left. 3\, z^{10} - 4\, u\, z^{10} + u^2\, z^{10} + 2\, z^{11} - 4\, u\, z^{11} + 2\, u^2\, z^{11} + z^{12} - 2\, u\, z^{12} + u^2\, z^{12}\right)$$

$$4 - 2\, z^2 - 8\, s\, z^2 + 4\, z^3 + 4\, z^4 + 2\, s\, z^4 + 3\, s^2\, z^4 + 2\, z^5 - 4\, s\, z^5 - 4\, u\, z^5 +$$
$$3\, z^6 - 4\, s\, z^6 - 4\, u\, z^6 - 4\, s\, z^7 + 2\, u\, z^7 + 4\, s\, u\, z^7 + z^8 - 4\, s\, z^8 + 4\, s\, u\, z^8 + 4\, z^9 -$$
$$4\, u\, z^9 + 3\, z^{10} - 4\, u\, z^{10} + u^2\, z^{10} + 2\, z^{11} - 4\, u\, z^{11} + 2\, u^2\, z^{11} + z^{12} - 2\, u\, z^{12} + u^2\, z^{12}$$

Denominator of (s-f) same as that of (1-dfs)

Now compute the resultant of numerators a,b to get relation between S,u,z

Express S-F and 1-D[F,S] as fractions over the same common denominator

Then compute the resultant of the numerators of these expressions

$$-128\, z^{11} - 144\, z^{12} - 32\, z^{13} + 224\, z^{14} + 608\, z^{15} + 796\, z^{16} + 192\, u\, z^{16} + 776\, z^{17} + 400\, u\, z^{17} +$$
$$244\, z^{18} + 448\, u\, z^{18} - 464\, z^{19} + 680\, u\, z^{19} - 1204\, z^{20} + 928\, u\, z^{20} - 1808\, z^{21} + 1264\, u\, z^{21} -$$
$$96\, u^2\, z^{21} - 1996\, z^{22} + 1840\, u\, z^{22} - 292\, u^2\, z^{22} - 1960\, z^{23} + 2128\, u\, z^{23} - 440\, u^2\, z^{23} - 1700\, z^{24} +$$
$$2120\, u\, z^{24} - 580\, u^2\, z^{24} - 1408\, z^{25} + 2016\, u\, z^{25} - 672\, u^2\, z^{25} - 1088\, z^{26} + 1728\, u\, z^{26} - 672\, u^2\, z^{26} +$$
$$16\, u^3\, z^{26} - 784\, z^{27} + 1440\, u\, z^{27} - 720\, u^2\, z^{27} + 64\, u^3\, z^{27} - 544\, z^{28} + 1152\, u\, z^{28} - 720\, u^2\, z^{28} +$$
$$112\, u^3\, z^{28} - 320\, z^{29} + 768\, u\, z^{29} - 576\, u^2\, z^{29} + 128\, u^3\, z^{29} - 160\, z^{30} + 432\, u\, z^{30} - 384\, u^2\, z^{30} +$$
$$112\, u^3\, z^{30} - 64\, z^{31} + 192\, u\, z^{31} - 192\, u^2\, z^{31} + 64\, u^3\, z^{31} - 16\, z^{32} + 48\, u\, z^{32} - 48\, u^2\, z^{32} + 16\, u^3\, z^{32}$$

$$-128\, z[u]^{11} - 144\, z[u]^{12} - 32\, z[u]^{13} + 224\, z[u]^{14} + 608\, z[u]^{15} + 796\, z[u]^{16} + 192\, u\, z[u]^{16} +$$
$$776\, z[u]^{17} + 400\, u\, z[u]^{17} + 244\, z[u]^{18} + 448\, u\, z[u]^{18} - 464\, z[u]^{19} + 680\, u\, z[u]^{19} - 1204\, z[u]^{20} +$$
$$928\, u\, z[u]^{20} - 1808\, z[u]^{21} + 1264\, u\, z[u]^{21} - 96\, u^2\, z[u]^{21} - 1996\, z[u]^{22} + 1840\, u\, z[u]^{22} -$$
$$292\, u^2\, z[u]^{22} - 1960\, z[u]^{23} + 2128\, u\, z[u]^{23} - 440\, u^2\, z[u]^{23} - 1700\, z[u]^{24} + 2120\, u\, z[u]^{24} -$$
$$580\, u^2\, z[u]^{24} - 1408\, z[u]^{25} + 2016\, u\, z[u]^{25} - 672\, u^2\, z[u]^{25} - 1088\, z[u]^{26} + 1728\, u\, z[u]^{26} -$$
$$672\, u^2\, z[u]^{26} + 16\, u^3\, z[u]^{26} - 784\, z[u]^{27} + 1440\, u\, z[u]^{27} - 720\, u^2\, z[u]^{27} + 64\, u^3\, z[u]^{27} -$$
$$544\, z[u]^{28} + 1152\, u\, z[u]^{28} - 720\, u^2\, z[u]^{28} + 112\, u^3\, z[u]^{28} - 320\, z[u]^{29} + 768\, u\, z[u]^{29} -$$
$$576\, u^2\, z[u]^{29} + 128\, u^3\, z[u]^{29} - 160\, z[u]^{30} + 432\, u\, z[u]^{30} - 384\, u^2\, z[u]^{30} + 112\, u^3\, z[u]^{30} - 64\, z[u]^{31} +$$
$$192\, u\, z[u]^{31} - 192\, u^2\, z[u]^{31} + 64\, u^3\, z[u]^{31} - 16\, z[u]^{32} + 48\, u\, z[u]^{32} - 48\, u^2\, z[u]^{32} + 16\, u^3\, z[u]^{32}$$

$192\, z[u]^{16} + 400\, z[u]^{17} + 448\, z[u]^{18} + 680\, z[u]^{19} + 928\, z[u]^{20} + 1264\, z[u]^{21} - 192\, u\, z[u]^{21} + 1840\, z[u]^{22} - 584\, u\, z[u]^{22} + 2128\, z[u]^{23} - 880\, u\, z[u]^{23} + 2120\, z[u]^{24} - 1160\, u\, z[u]^{24} + 2016\, z[u]^{25} - 1344\, u\, z[u]^{25} + 1728\, z[u]^{26} - 1344\, u\, z[u]^{26} + 48\, u^2\, z[u]^{26} + 1440\, z[u]^{27} - 1440\, u\, z[u]^{27} + 192\, u^2\, z[u]^{27} + 1152\, z[u]^{28} - 1440\, u\, z[u]^{28} + 336\, u^2\, z[u]^{28} + 768\, z[u]^{29} - 1152\, u\, z[u]^{29} + 384\, u^2\, z[u]^{29} + 432\, z[u]^{30} - 768\, u\, z[u]^{30} + 336\, u^2\, z[u]^{30} + 192\, z[u]^{31} - 384\, u\, z[u]^{31} + 192\, u^2\, z[u]^{31} + 48\, z[u]^{32} - 96\, u\, z[u]^{32} + 48\, u^2\, z[u]^{32} - 1408\, z[u]^{10}\, \mathrm{Derivative}[1][z][u] - 1728\, z[u]^{11}\, \mathrm{Derivative}[1][z][u] - 416\, z[u]^{12}\, \mathrm{Derivative}[1][z][u] + 3136\, z[u]^{13}\, \mathrm{Derivative}[1][z][u] + 9120\, z[u]^{14}\, \mathrm{Derivative}[1][z][u] + 12\,736\, z[u]^{15}\, \mathrm{Derivative}[1][z][u] + 3072\, u\, z[u]^{15}\, \mathrm{Derivative}[1][z][u] + 13\,192\, z[u]^{16}\, \mathrm{Derivative}[1][z][u] + 6800\, u\, z[u]^{16}\, \mathrm{Derivative}[1][z][u] + 4392\, z[u]^{17}\, \mathrm{Derivative}[1][z][u] + 8064\, u\, z[u]^{17}\, \mathrm{Derivative}[1][z][u] - 8816\, z[u]^{18}\, \mathrm{Derivative}[1][z][u] + 12\,920\, u\, z[u]^{18}\, \mathrm{Derivative}[1][z][u] - 24\,080\, z[u]^{19}\, \mathrm{Derivative}[1][z][u] + 18\,560\, u\, z[u]^{19}\, \mathrm{Derivative}[1][z][u] - 37\,968\, z[u]^{20}\, \mathrm{Derivative}[1][z][u] + 26\,544\, u\, z[u]^{20}\, \mathrm{Derivative}[1][z][u] - 2016\, u^2\, z[u]^{20}\, \mathrm{Derivative}[1][z][u] - 43\,912\, z[u]^{21}\, \mathrm{Derivative}[1][z][u] + 40\,480\, u\, z[u]^{21}\, \mathrm{Derivative}[1][z][u] - 6424\, u^2\, z[u]^{21}\, \mathrm{Derivative}[1][z][u] - 45\,080\, z[u]^{22}\, \mathrm{Derivative}[1][z][u] + 48\,944\, u\, z[u]^{22}\, \mathrm{Derivative}[1][z][u] - 10\,120\, u^2\, z[u]^{22}\, \mathrm{Derivative}[1][z][u] - 40\,800\, z[u]^{23}\, \mathrm{Derivative}[1][z][u] + 50\,880\, u\, z[u]^{23}\, \mathrm{Derivative}[1][z][u] - 13\,920\, u^2\, z[u]^{23}\, \mathrm{Derivative}[1][z][u] - 35\,200\, z[u]^{24}\, \mathrm{Derivative}[1][z][u] + 50\,400\, u\, z[u]^{24}\, \mathrm{Derivative}[1][z][u] - 16\,800\, u^2\, z[u]^{24}\, \mathrm{Derivative}[1][z][u] - 28\,288\, z[u]^{25}\, \mathrm{Derivative}[1][z][u] + 44\,928\, u\, z[u]^{25}\, \mathrm{Derivative}[1][z][u] - 17\,472\, u^2\, z[u]^{25}\, \mathrm{Derivative}[1][z][u] + 416\, u^3\, z[u]^{25}\, \mathrm{Derivative}[1][z][u] - 21\,168\, z[u]^{26}\, \mathrm{Derivative}[1][z][u] + 38\,880\, u\, z[u]^{26}\, \mathrm{Derivative}[1][z][u] - 19\,440\, u^2\, z[u]^{26}\, \mathrm{Derivative}[1][z][u] + 1728\, u^3\, z[u]^{26}\, \mathrm{Derivative}[1][z][u] - 15\,232\, z[u]^{27}\, \mathrm{Derivative}[1][z][u] + 32\,256\, u\, z[u]^{27}\, \mathrm{Derivative}[1][z][u] - 20\,160\, u^2\, z[u]^{27}\, \mathrm{Derivative}[1][z][u] + 3136\, u^3\, z[u]^{27}\, \mathrm{Derivative}[1][z][u] - 9280\, z[u]^{28}\, \mathrm{Derivative}[1][z][u] + 22\,272\, u\, z[u]^{28}\, \mathrm{Derivative}[1][z][u] - 16\,704\, u^2\, z[u]^{28}\, \mathrm{Derivative}[1][z][u] + 3712\, u^3\, z[u]^{28}\, \mathrm{Derivative}[1][z][u] - 4800\, z[u]^{29}\, \mathrm{Derivative}[1][z][u] + 12\,960\, u\, z[u]^{29}\, \mathrm{Derivative}[1][z][u] - 11\,520\, u^2\, z[u]^{29}\, \mathrm{Derivative}[1][z][u] + 3360\, u^3\, z[u]^{29}\, \mathrm{Derivative}[1][z][u] - 1984\, z[u]^{30}\, \mathrm{Derivative}[1][z][u] + 5952\, u\, z[u]^{30}\, \mathrm{Derivative}[1][z][u] - 5952\, u^2\, z[u]^{30}\, \mathrm{Derivative}[1][z][u] + 1984\, u^3\, z[u]^{30}\, \mathrm{Derivative}[1][z][u] - 512\, z[u]^{31}\, \mathrm{Derivative}[1][z][u] + 1536\, u\, z[u]^{31}\, \mathrm{Derivative}[1][z][u] - 1536\, u^2\, z[u]^{31}\, \mathrm{Derivative}[1][z][u] + 512\, u^3\, z[u]^{31}\, \mathrm{Derivative}[1][z][u]$

$8 z[u]^{10} \left(6 (-1+u)^2 z[u]^{22} - 176 \text{ Derivative}[1][z][u] - 216 z[u] \text{ Derivative}[1][z][u] - \right.$

$\quad 52 z[u]^2 \text{ Derivative}[1][z][u] + 392 z[u]^3 \text{ Derivative}[1][z][u] +$

$\quad 1140 z[u]^4 \text{ Derivative}[1][z][u] + 8 (199 + 48 u) z[u]^5 \text{ Derivative}[1][z][u] +$

$\quad 8 (-1+u)^2 z[u]^{21} (3 + 8 (-1+u) \text{ Derivative}[1][z][u]) +$

$\quad 2 (-1+u) z[u]^{20} \left(3 (-9 + 7 u) + 124 (-1+u)^2 \text{ Derivative}[1][z][u]\right) +$

$\quad 2 z[u]^{18} \left(72 - 90 u + 21 u^2 + 116 (-1+u)^2 (-5 + 2 u) \text{ Derivative}[1][z][u]\right) +$

$\quad z[u]^6 (24 + 17 (97 + 50 u) \text{ Derivative}[1][z][u]) +$

$\quad z[u]^8 (56 + 19 (-58 + 85 u) \text{ Derivative}[1][z][u]) +$

$\quad z[u]^7 (50 + 9 (61 + 112 u) \text{ Derivative}[1][z][u]) +$

$\quad 5 z[u]^9 (17 + (-602 + 464 u) \text{ Derivative}[1][z][u]) +$

$\quad z[u]^{10} \left(116 - 42 (113 - 79 u + 6 u^2) \text{ Derivative}[1][z][u]\right) +$

$\quad 12 (-1+u) z[u]^{19} \left(4 (-2+u) + 5 (10 - 17 u + 7 u^2) \text{ Derivative}[1][z][u]\right) -$

$\quad 5 z[u]^{14} \left(-53 + 29 u + 20 (44 - 63 u + 21 u^2) \text{ Derivative}[1][z][u]\right) -$

$\quad 2 z[u]^{13} \left(-133 + 55 u + 30 (85 - 106 u + 29 u^2) \text{ Derivative}[1][z][u]\right) +$

$\quad z[u]^{12} \left(230 - 73 u - 23 (245 - 266 u + 55 u^2) \text{ Derivative}[1][z][u]\right) +$

$\quad z[u]^{11} \left(158 - 24 u - 11 (499 - 460 u + 73 u^2) \text{ Derivative}[1][z][u]\right) +$

$\quad 4 z[u]^{15} \left(63 - 42 u + 13 (-68 + 108 u - 42 u^2 + u^3) \text{ Derivative}[1][z][u]\right) +$

$\quad 6 z[u]^{16} \left(36 - 28 u + u^2 + 9 (-49 + 90 u - 45 u^2 + 4 u^3) \text{ Derivative}[1][z][u]\right) +$

$\quad \left. 4 z[u]^{17} \left(45 - 45 u + 6 u^2 + 14 (-34 + 72 u - 45 u^2 + 7 u^3) \text{ Derivative}[1][z][u]\right)\right)$

$\left\{\left\{\text{Derivative}[1][z][u] \rightarrow \right.\right.$

$\quad \left(-24 z[u]^6 - 50 z[u]^7 - 56 z[u]^8 - 85 z[u]^9 - 116 z[u]^{10} - 158 z[u]^{11} + 24 u z[u]^{11} - 230 z[u]^{12} + \right.$

$\qquad 73 u z[u]^{12} - 266 z[u]^{13} + 110 u z[u]^{13} - 265 z[u]^{14} + 145 u z[u]^{14} - 252 z[u]^{15} + 168 u z[u]^{15} -$

$\qquad 216 z[u]^{16} + 168 u z[u]^{16} - 6 u^2 z[u]^{16} - 180 z[u]^{17} + 180 u z[u]^{17} - 24 u^2 z[u]^{17} - 144 z[u]^{18} +$

$\qquad 180 u z[u]^{18} - 42 u^2 z[u]^{18} - 96 z[u]^{19} + 144 u z[u]^{19} - 48 u^2 z[u]^{19} - 54 z[u]^{20} + 96 u z[u]^{20} -$

$\qquad \left. 42 u^2 z[u]^{20} - 24 z[u]^{21} + 48 u z[u]^{21} - 24 u^2 z[u]^{21} - 6 z[u]^{22} + 12 u z[u]^{22} - 6 u^2 z[u]^{22}\right) \Big/$

$\qquad \left(-176 - 216 z[u] - 52 z[u]^2 + 392 z[u]^3 + 1140 z[u]^4 + 1592 z[u]^5 + 384 u z[u]^5 + \right.$

$\qquad 1649 z[u]^6 + 850 u z[u]^6 + 549 z[u]^7 + 1008 u z[u]^7 - 1102 z[u]^8 + 1615 u z[u]^8 -$

$\qquad 3010 z[u]^9 + 2320 u z[u]^9 - 4746 z[u]^{10} + 3318 u z[u]^{10} - 252 u^2 z[u]^{10} - 5489 z[u]^{11} +$

$\qquad 5060 u z[u]^{11} - 803 u^2 z[u]^{11} - 5635 z[u]^{12} + 6118 u z[u]^{12} - 1265 u^2 z[u]^{12} - 5100 z[u]^{13} +$

$\qquad 6360 u z[u]^{13} - 1740 u^2 z[u]^{13} - 4400 z[u]^{14} + 6300 u z[u]^{14} - 2100 u^2 z[u]^{14} -$

$\qquad 3536 z[u]^{15} + 5616 u z[u]^{15} - 2184 u^2 z[u]^{15} + 52 u^3 z[u]^{15} - 2646 z[u]^{16} + 4860 u z[u]^{16} -$

$\qquad 2430 u^2 z[u]^{16} + 216 u^3 z[u]^{16} - 1904 z[u]^{17} + 4032 u z[u]^{17} - 2520 u^2 z[u]^{17} +$

$\qquad 392 u^3 z[u]^{17} - 1160 z[u]^{18} + 2784 u z[u]^{18} - 2088 u^2 z[u]^{18} + 464 u^3 z[u]^{18} -$

$\qquad 600 z[u]^{19} + 1620 u z[u]^{19} - 1440 u^2 z[u]^{19} + 420 u^3 z[u]^{19} - 248 z[u]^{20} + 744 u z[u]^{20} -$

$\qquad \left.\left.\left. 744 u^2 z[u]^{20} + 248 u^3 z[u]^{20} - 64 z[u]^{21} + 192 u z[u]^{21} - 192 u^2 z[u]^{21} + 64 u^3 z[u]^{21}\right)\right\}\right\}$

$-0.0373225$

According to Drmota's Theorem 1, the mean equals -z'[1]/z[1], computed next.

$0.0651437$

$0.0289688$

Now compute variance, which by Drmota is -z''[1]/z[1] + mu^2 + mu

$0.0188246$