(∗ Asymptotic expected number of hairpins in saturated structures with theta= 3 and stickiness p=3/8∗)

(∗We first compute the dominant singularity and
 asymptotic number of saturated structures,
using Drmota−Lalley−Woods Theorem. ∗)

```
Clear["*"]
Clear[p, S, D0, N0, z, R, eqn0, eqn, F, z0, y0, dFdzOfz0S0, d2FdyOfz0S0];
p = 3 / 8;
eqn = {S == D0 + N0, D0 == z + z^2 + z^3 + z^4,
    N0 == R D0 +  p (z^3 + z^4) z^2 + p N0 z^2 + p S (z^3 + z^4) z^2 + p S N0 z^2,
    R == p (z^3 + z^4) z^2 + p N0 z^2 + p R (z^3 + z^4) z^2 + p R N0 z^2};
Eliminate[eqn, {N0, D0, R}]
```

$$F = \left(9\, S^3\, z^4 + S^2\, z^2 \left(-48 + 9\, z^2 - 18\, z^3 - 18\, z^4\right) - z \left(64 + 64\, z + 40\, z^2 + 40\, z^3 - 9\, z^5 - 18\, z^6 - 9\, z^7\right)\right) /$$
$$\left(-\left(64 - 24\, z^2 + 48\, z^3 + 48\, z^4 - 18\, z^5 - 9\, z^6 + 18\, z^7 + 9\, z^8\right)\right)$$

```
NSolve[{F == S, D[F, S] == 1}, {z, S}];
z0 = 0.6166066469444536`
y0 = 2.4218598552257076`

dFdzOfz0S0 = D[F, z] /. {z → z0, S → y0}

d2FdyOfz0S0 = D[F, {S, 2}] /. {z → z0, S → y0}
c = Sqrt[z0 dFdzOfz0S0 / (2 Pi d2FdyOfz0S0)]
c * (1 / z0) ^n n^ (-3 / 2)
```

$$9\, S^3\, z^4 + S^2\, z^2 \left(-48 + 9\, z^2 - 18\, z^3 - 18\, z^4\right) + S \left(64 - 24\, z^2 + 48\, z^3 + 48\, z^4 - 18\, z^5 - 9\, z^6 + 18\, z^7 + 9\, z^8\right) ==$$
$$z \left(64 + 64\, z + 40\, z^2 + 40\, z^3 - 9\, z^5 - 18\, z^6 - 9\, z^7\right)$$

$$\frac{9\, S^3\, z^4 + S^2\, z^2 \left(-48 + 9\, z^2 - 18\, z^3 - 18\, z^4\right) - z \left(64 + 64\, z + 40\, z^2 + 40\, z^3 - 9\, z^5 - 18\, z^6 - 9\, z^7\right)}{-64 + 24\, z^2 - 48\, z^3 - 48\, z^4 + 18\, z^5 + 9\, z^6 - 18\, z^7 - 9\, z^8}$$

0.616607

2.42186

5.19773

0.281193

1.34685

$$\frac{1.34685 \times 1.62178^n}{n^{3/2}}$$

(∗Now, we compute mean,variance using Drmota's Theorem∗)

```mathematica
Clear["*"]
p = 3 / 8;
eqn = {S == D0 + N0, D0 == z + z^2 + z^3 + z^4,
    N0 == R D0 +  p u (z^3 + z^4) z^2 + p N0 z^2 + p u S (z^3 + z^4) z^2 + p S N0 z^2,
    R == p u (z^3 + z^4) z^2 + p N0 z^2 + p u R (z^3 + z^4) z^2 + p R N0 z^2};

CellPrint["Eliminate all variables except S,u,z"]
Eliminate[eqn, {D0, N0, R}]
Collect[%, S, Simplify]
```

$$F = \left(9\,S^3\,z^4 + 3\,S^2\,z^2\,\left(-16 + 3\,z^2 - 6\,z^3 - 6\,z^4 + 6\,(-1 + u)\,z^5 + 6\,(-1 + u)\,z^6\right) + z\,(1 + z)\,\left(-64 - 40\,z^2 - \right.\right.$$
$$\left.\left.24\,(-1 + u)\,z^4 + 9\,z^5 + 9\,z^6 - 18\,(-1 + u)\,z^7 - 18\,(-1 + u)\,z^8 + 9\,(-1 + u)^2\,z^9 + 9\,(-1 + u)^2\,z^{10}\right)\right) \Big/$$
$$\left(-\left(64 - 24\,z^2 + 48\,z^3 + 48\,z^4 - 6\,(-5 + 8\,u)\,z^5 + (39 - 48\,u)\,z^6 + 18\,u\,z^7 + 9\,z^8 - \right.\right.$$
$$\left.\left.36\,(-1 + u)\,z^9 + 9\,\left(3 - 4\,u + u^2\right)\,z^{10} + 18\,(-1 + u)^2\,z^{11} + 9\,(-1 + u)^2\,z^{12}\right)\right)$$

```mathematica
f = (F /. S → s)
s - f
(* express over a common denominator*)
Together[s - f]
a = Numerator[%]

dfs = D[f, s]
1 - dfs
(* express over a common denominator*)
Together[1 - dfs]
(* a is numerator of s-f, and b is numerator of 1-dfs,
where both have identical denominators *)
b = Numerator[%]
(* a is numerator of s-f, and b is numerator of 1-dfs,
where both have identical denominators *)
If[Denominator[Together[s - f]] == Denominator[Together[1 - dfs]],
 CellPrint["Denominator of (s-f) same as that of (1-dfs)"],
 CellPrint["Denominator of (s-f) different than than of (1-dfs)"]]
CellPrint["Now compute the resultant of numerators a,b to get relation between S,u,z"]

(* res =Resultant[s-f,1-dfs,s] *)


(*We compute resultant of numerators,
since we have Resultant[S-F,1-D[F,S],S]=0 which is not much use.*)
CellPrint["Express S-F and 1-D[F,S] as fractions over the same common denominator"]
CellPrint["Then compute the resultant of the numerators of these expressions"]
res = Resultant[a, b, s]
(* Replace z by z[u], a function of u *)


res /. z → z[u]
(* Now compute z'[u] *)
dres = D[%, u]
Simplify[Collect[dres, z'[u]]]
Solve[dres == 0, z'[u]]
dzu = Last[Last[Last[Solve[dres == 0, z'[u]]]]];
(*z[1] equals rho, the dominant singularity *)




rho = 0.6166066469444536`;
(* value of z0 in the first part of this file, the dominant singularity*)
dzuEvaluatedAt1 = (dzu /. u → 1) /. z[1] → rho
CellPrint[
 "According to Drmota's Theorem 1, the mean equals -z'[1]/z[1], computed next. "]
```

```
mu = ((-dzu / z[u] /. u → 1) /. z[1] → rho)

(*Variance computation *)
(* d2zu is z''[1] *)
d2zu = (((D[dzu, u] /. u → 1) /. z[1] → rho) /. z'[1] → dzuEvaluatedAt1)
CellPrint["Now compute variance, which by Drmota is -z''[1]/z[1] + mu^2 + mu"]
var = -d2zu / rho + mu * mu + mu
```

Eliminate all variables except S,u,z

$$9\,S^3\,z^4 + S^2\,z^2\,\left(-48 + 9\,z^2 - 18\,z^3 - 18\,z^4 - 18\,z^5 + 18\,u\,z^5 - 18\,z^6 + 18\,u\,z^6\right) +$$
$$S\,\left(64 - 24\,z^2 + 48\,z^3 + 48\,z^4 + 30\,z^5 - 48\,u\,z^5 + 39\,z^6 - 48\,u\,z^6 + 18\,u\,z^7 + 9\,z^8 + 36\,z^9 - 36\,u\,z^9 +\right.$$
$$\left.27\,z^{10} - 36\,u\,z^{10} + 9\,u^2\,z^{10} + 18\,z^{11} - 36\,u\,z^{11} + 18\,u^2\,z^{11} + 9\,z^{12} - 18\,u\,z^{12} + 9\,u^2\,z^{12}\right) ==$$
$$z\,\left(64 + 64\,z + 40\,z^2 + 40\,z^3 - 24\,z^4 + 24\,u\,z^4 - 33\,z^5 + 24\,u\,z^5 - 18\,z^6 - 27\,z^7 + 18\,u\,z^7 - 36\,z^8 +\right.$$
$$\left.36\,u\,z^8 - 27\,z^9 + 36\,u\,z^9 - 9\,u^2\,z^9 - 18\,z^{10} + 36\,u\,z^{10} - 18\,u^2\,z^{10} - 9\,z^{11} + 18\,u\,z^{11} - 9\,u^2\,z^{11}\right)$$

$$9\,S^3\,z^4 + 3\,S^2\,z^2\,\left(-16 + 3\,z^2 - 6\,z^3 - 6\,z^4 + 6\,(-1 + u)\,z^5 + 6\,(-1 + u)\,z^6\right) +$$
$$S\,\left(64 - 24\,z^2 + 48\,z^3 + 48\,z^4 - 6\,(-5 + 8\,u)\,z^5 + (39 - 48\,u)\,z^6 + 18\,u\,z^7 +\right.$$
$$\left.9\,z^8 - 36\,(-1 + u)\,z^9 + 9\,\left(3 - 4\,u + u^2\right)\,z^{10} + 18\,(-1 + u)^2\,z^{11} + 9\,(-1 + u)^2\,z^{12}\right) ==$$
$$-z\,(1 + z)\,\left(-64 - 40\,z^2 - 24\,(-1 + u)\,z^4 + 9\,z^5 + 9\,z^6 - 18\,(-1 + u)\,z^7 -\right.$$
$$\left.18\,(-1 + u)\,z^8 + 9\,(-1 + u)^2\,z^9 + 9\,(-1 + u)^2\,z^{10}\right)$$

$$\left(9\,S^3\,z^4 + 3\,S^2\,z^2\,\left(-16 + 3\,z^2 - 6\,z^3 - 6\,z^4 + 6\,(-1 + u)\,z^5 + 6\,(-1 + u)\,z^6\right) +\right.$$
$$z\,(1 + z)\,\left(-64 - 40\,z^2 - 24\,(-1 + u)\,z^4 + 9\,z^5 + 9\,z^6 -\right.$$
$$\left.\left.18\,(-1 + u)\,z^7 - 18\,(-1 + u)\,z^8 + 9\,(-1 + u)^2\,z^9 + 9\,(-1 + u)^2\,z^{10}\right)\right) /$$
$$\left(-64 + 24\,z^2 - 48\,z^3 - 48\,z^4 + 6\,(-5 + 8\,u)\,z^5 - (39 - 48\,u)\,z^6 - 18\,u\,z^7 - 9\,z^8 +\right.$$
$$\left.36\,(-1 + u)\,z^9 - 9\,\left(3 - 4\,u + u^2\right)\,z^{10} - 18\,(-1 + u)^2\,z^{11} - 9\,(-1 + u)^2\,z^{12}\right)$$

$$\left(9\,s^3\,z^4 + 3\,s^2\,z^2\,\left(-16 + 3\,z^2 - 6\,z^3 - 6\,z^4 + 6\,(-1 + u)\,z^5 + 6\,(-1 + u)\,z^6\right) +\right.$$
$$z\,(1 + z)\,\left(-64 - 40\,z^2 - 24\,(-1 + u)\,z^4 + 9\,z^5 + 9\,z^6 -\right.$$
$$\left.\left.18\,(-1 + u)\,z^7 - 18\,(-1 + u)\,z^8 + 9\,(-1 + u)^2\,z^9 + 9\,(-1 + u)^2\,z^{10}\right)\right) /$$
$$\left(-64 + 24\,z^2 - 48\,z^3 - 48\,z^4 + 6\,(-5 + 8\,u)\,z^5 - (39 - 48\,u)\,z^6 - 18\,u\,z^7 - 9\,z^8 +\right.$$
$$\left.36\,(-1 + u)\,z^9 - 9\,\left(3 - 4\,u + u^2\right)\,z^{10} - 18\,(-1 + u)^2\,z^{11} - 9\,(-1 + u)^2\,z^{12}\right)$$

$$s - \left(9\,s^3\,z^4 + 3\,s^2\,z^2\,\left(-16 + 3\,z^2 - 6\,z^3 - 6\,z^4 + 6\,(-1 + u)\,z^5 + 6\,(-1 + u)\,z^6\right) + z\,(1 + z)\,\left(-64 - 40\,z^2 -\right.\right.$$
$$\left.\left.24\,(-1 + u)\,z^4 + 9\,z^5 + 9\,z^6 - 18\,(-1 + u)\,z^7 - 18\,(-1 + u)\,z^8 + 9\,(-1 + u)^2\,z^9 + 9\,(-1 + u)^2\,z^{10}\right)\right) /$$
$$\left(-64 + 24\,z^2 - 48\,z^3 - 48\,z^4 + 6\,(-5 + 8\,u)\,z^5 - (39 - 48\,u)\,z^6 - 18\,u\,z^7 - 9\,z^8 +\right.$$
$$\left.36\,(-1 + u)\,z^9 - 9\,\left(3 - 4\,u + u^2\right)\,z^{10} - 18\,(-1 + u)^2\,z^{11} - 9\,(-1 + u)^2\,z^{12}\right)$$

$$\left(64\,s - 64\,z - 64\,z^2 - 24\,s\,z^2 - 48\,s^2\,z^2 - 40\,z^3 + 48\,s\,z^3 - 40\,z^4 + 48\,s\,z^4 + 9\,s^2\,z^4 + 9\,s^3\,z^4 + 24\,z^5 + 30\,s\,z^5 -\right.$$
$$18\,s^2\,z^5 - 24\,u\,z^5 - 48\,s\,u\,z^5 + 33\,z^6 + 39\,s\,z^6 - 18\,s^2\,z^6 - 24\,u\,z^6 - 48\,s\,u\,z^6 + 18\,z^7 - 18\,s^2\,z^7 +$$
$$18\,s\,u\,z^7 + 18\,s^2\,u\,z^7 + 27\,z^8 + 9\,s\,z^8 - 18\,s^2\,z^8 - 18\,u\,z^8 + 18\,s^2\,u\,z^8 + 36\,z^9 + 36\,s\,z^9 - 36\,u\,z^9 -$$
$$36\,s\,u\,z^9 + 27\,z^{10} + 27\,s\,z^{10} - 36\,u\,z^{10} - 36\,s\,u\,z^{10} + 9\,u^2\,z^{10} + 9\,s\,u^2\,z^{10} + 18\,z^{11} + 18\,s\,z^{11} - 36\,u\,z^{11} -$$
$$\left.36\,s\,u\,z^{11} + 18\,u^2\,z^{11} + 18\,s\,u^2\,z^{11} + 9\,z^{12} + 9\,s\,z^{12} - 18\,u\,z^{12} - 18\,s\,u\,z^{12} + 9\,u^2\,z^{12} + 9\,s\,u^2\,z^{12}\right) /$$
$$\left(64 - 24\,z^2 + 48\,z^3 + 48\,z^4 + 30\,z^5 - 48\,u\,z^5 + 39\,z^6 - 48\,u\,z^6 + 18\,u\,z^7 + 9\,z^8 + 36\,z^9 - 36\,u\,z^9 +\right.$$
$$\left.27\,z^{10} - 36\,u\,z^{10} + 9\,u^2\,z^{10} + 18\,z^{11} - 36\,u\,z^{11} + 18\,u^2\,z^{11} + 9\,z^{12} - 18\,u\,z^{12} + 9\,u^2\,z^{12}\right)$$

$$64\,s - 64\,z - 64\,z^2 - 24\,s\,z^2 - 48\,s^2\,z^2 - 40\,z^3 + 48\,s\,z^3 - 40\,z^4 + 48\,s\,z^4 + 9\,s^2\,z^4 + 9\,s^3\,z^4 + 24\,z^5 + 30\,s\,z^5 -$$
$$18\,s^2\,z^5 - 24\,u\,z^5 - 48\,s\,u\,z^5 + 33\,z^6 + 39\,s\,z^6 - 18\,s^2\,z^6 - 24\,u\,z^6 - 48\,s\,u\,z^6 + 18\,z^7 - 18\,s^2\,z^7 +$$
$$18\,s\,u\,z^7 + 18\,s^2\,u\,z^7 + 27\,z^8 + 9\,s\,z^8 - 18\,s^2\,z^8 - 18\,u\,z^8 + 18\,s^2\,u\,z^8 + 36\,z^9 + 36\,s\,z^9 - 36\,u\,z^9 -$$
$$36\,s\,u\,z^9 + 27\,z^{10} + 27\,s\,z^{10} - 36\,u\,z^{10} - 36\,s\,u\,z^{10} + 9\,u^2\,z^{10} + 9\,s\,u^2\,z^{10} + 18\,z^{11} + 18\,s\,z^{11} - 36\,u\,z^{11} -$$
$$36\,s\,u\,z^{11} + 18\,u^2\,z^{11} + 18\,s\,u^2\,z^{11} + 9\,z^{12} + 9\,s\,z^{12} - 18\,u\,z^{12} - 18\,s\,u\,z^{12} + 9\,u^2\,z^{12} + 9\,s\,u^2\,z^{12}$$

$$\left(27\, s^2\, z^4 + 6\, s\, z^2\, \left(-16 + 3\, z^2 - 6\, z^3 - 6\, z^4 + 6\, (-1+u)\, z^5 + 6\, (-1+u)\, z^6\right)\right) \big/$$
$$\left(-64 + 24\, z^2 - 48\, z^3 - 48\, z^4 + 6\, (-5 + 8\, u)\, z^5 - (39 - 48\, u)\, z^6 - 18\, u\, z^7 - \right.$$
$$\left. 9\, z^8 + 36\, (-1+u)\, z^9 - 9\, \left(3 - 4\, u + u^2\right) z^{10} - 18\, (-1+u)^2\, z^{11} - 9\, (-1+u)^2\, z^{12}\right)$$

$$1 - \left(27\, s^2\, z^4 + 6\, s\, z^2\, \left(-16 + 3\, z^2 - 6\, z^3 - 6\, z^4 + 6\, (-1+u)\, z^5 + 6\, (-1+u)\, z^6\right)\right) \big/$$
$$\left(-64 + 24\, z^2 - 48\, z^3 - 48\, z^4 + 6\, (-5 + 8\, u)\, z^5 - (39 - 48\, u)\, z^6 - 18\, u\, z^7 - \right.$$
$$\left. 9\, z^8 + 36\, (-1+u)\, z^9 - 9\, \left(3 - 4\, u + u^2\right) z^{10} - 18\, (-1+u)^2\, z^{11} - 9\, (-1+u)^2\, z^{12}\right)$$

$$\left(64 - 24\, z^2 - 96\, s\, z^2 + 48\, z^3 + 48\, z^4 + 18\, s\, z^4 + 27\, s^2\, z^4 + 30\, z^5 - 36\, s\, z^5 - 48\, u\, z^5 + 39\, z^6 - \right.$$
$$36\, s\, z^6 - 48\, u\, z^6 - 36\, s\, z^7 + 18\, u\, z^7 + 36\, s\, u\, z^7 + 9\, z^8 - 36\, s\, z^8 + 36\, s\, u\, z^8 + 36\, z^9 - 36\, u\, z^9 +$$
$$\left. 27\, z^{10} - 36\, u\, z^{10} + 9\, u^2\, z^{10} + 18\, z^{11} - 36\, u\, z^{11} + 18\, u^2\, z^{11} + 9\, z^{12} - 18\, u\, z^{12} + 9\, u^2\, z^{12}\right) \big/$$
$$\left(64 - 24\, z^2 + 48\, z^3 + 48\, z^4 + 30\, z^5 - 48\, u\, z^5 + 39\, z^6 - 48\, u\, z^6 + 18\, u\, z^7 + 9\, z^8 + 36\, z^9 - 36\, u\, z^9 + \right.$$
$$\left. 27\, z^{10} - 36\, u\, z^{10} + 9\, u^2\, z^{10} + 18\, z^{11} - 36\, u\, z^{11} + 18\, u^2\, z^{11} + 9\, z^{12} - 18\, u\, z^{12} + 9\, u^2\, z^{12}\right)$$

$$64 - 24\, z^2 - 96\, s\, z^2 + 48\, z^3 + 48\, z^4 + 18\, s\, z^4 + 27\, s^2\, z^4 + 30\, z^5 - 36\, s\, z^5 - 48\, u\, z^5 +$$
$$39\, z^6 - 36\, s\, z^6 - 48\, u\, z^6 - 36\, s\, z^7 + 18\, u\, z^7 + 36\, s\, u\, z^7 + 9\, z^8 - 36\, s\, z^8 + 36\, s\, u\, z^8 + 36\, z^9 -$$
$$36\, u\, z^9 + 27\, z^{10} - 36\, u\, z^{10} + 9\, u^2\, z^{10} + 18\, z^{11} - 36\, u\, z^{11} + 18\, u^2\, z^{11} + 9\, z^{12} - 18\, u\, z^{12} + 9\, u^2\, z^{12}$$

Denominator of (s-f) same as that of (1-dfs)

Now compute the resultant of numerators a,b to get relation between S,u,z

Express S-F and 1-D[F,S] as fractions over the same common denominator

Then compute the resultant of the numerators of these expressions

$$-31\,850\,496\, z^{11} - 34\,836\,480\, z^{12} - 13\,934\,592\, z^{13} + 33\,094\,656\, z^{14} + 111\,974\,400\, z^{15} + 149\,625\,792\, z^{16} +$$
$$35\,831\,808\, u\, z^{16} + 154\,244\,736\, z^{17} + 73\,903\,104\, u\, z^{17} + 70\,403\,904\, z^{18} + 80\,621\,568\, u\, z^{18} - 36\,391\,680\, z^{19} +$$
$$112\,814\,208\, u\, z^{19} - 145\,426\,752\, z^{20} + 136\,608\,768\, u\, z^{20} - 233\,466\,624\, z^{21} + 169\,641\,216\, u\, z^{21} -$$
$$13\,436\,928\, u^2\, z^{21} - 251\,522\,496\, z^{22} + 241\,864\,704\, u\, z^{22} - 40\,730\,688\, u^2\, z^{22} - 239\,345\,280\, z^{23} +$$
$$270\,418\,176\, u\, z^{23} - 59\,626\,368\, u^2\, z^{23} - 198\,614\,592\, z^{24} + 256\,141\,440\, u\, z^{24} - 74\,323\,008\, u^2\, z^{24} -$$
$$157\,883\,904\, z^{25} + 231\,787\,008\, u\, z^{25} - 80\,621\,568\, u^2\, z^{25} - 117\,573\,120\, z^{26} + 188\,116\,992\, u\, z^{26} -$$
$$73\,903\,104\, u^2\, z^{26} + 1\,679\,616\, u^3\, z^{26} - 82\,301\,184\, z^{27} + 151\,165\,440\, u\, z^{27} - 75\,582\,720\, u^2\, z^{27} +$$
$$6\,718\,464\, u^3\, z^{27} - 57\,106\,944\, z^{28} + 120\,932\,352\, u\, z^{28} - 75\,582\,720\, u^2\, z^{28} + 11\,757\,312\, u^3\, z^{28} -$$
$$33\,592\,320\, z^{29} + 80\,621\,568\, u\, z^{29} - 60\,466\,176\, u^2\, z^{29} + 13\,436\,928\, u^3\, z^{29} - 16\,796\,160\, z^{30} +$$
$$45\,349\,632\, u\, z^{30} - 40\,310\,784\, u^2\, z^{30} + 11\,757\,312\, u^3\, z^{30} - 6\,718\,464\, z^{31} + 20\,155\,392\, u\, z^{31} -$$
$$20\,155\,392\, u^2\, z^{31} + 6\,718\,464\, u^3\, z^{31} - 1\,679\,616\, z^{32} + 5\,038\,848\, u\, z^{32} - 5\,038\,848\, u^2\, z^{32} + 1\,679\,616\, u^3\, z^{32}$$

$$-31\,850\,496\, z[u]^{11} - 34\,836\,480\, z[u]^{12} - 13\,934\,592\, z[u]^{13} + 33\,094\,656\, z[u]^{14} + 111\,974\,400\, z[u]^{15} +$$
$$149\,625\,792\, z[u]^{16} + 35\,831\,808\, u\, z[u]^{16} + 154\,244\,736\, z[u]^{17} + 73\,903\,104\, u\, z[u]^{17} +$$
$$70\,403\,904\, z[u]^{18} + 80\,621\,568\, u\, z[u]^{18} - 36\,391\,680\, z[u]^{19} + 112\,814\,208\, u\, z[u]^{19} -$$
$$145\,426\,752\, z[u]^{20} + 136\,608\,768\, u\, z[u]^{20} - 233\,466\,624\, z[u]^{21} + 169\,641\,216\, u\, z[u]^{21} -$$
$$13\,436\,928\, u^2\, z[u]^{21} - 251\,522\,496\, z[u]^{22} + 241\,864\,704\, u\, z[u]^{22} - 40\,730\,688\, u^2\, z[u]^{22} -$$
$$239\,345\,280\, z[u]^{23} + 270\,418\,176\, u\, z[u]^{23} - 59\,626\,368\, u^2\, z[u]^{23} - 198\,614\,592\, z[u]^{24} +$$
$$256\,141\,440\, u\, z[u]^{24} - 74\,323\,008\, u^2\, z[u]^{24} - 157\,883\,904\, z[u]^{25} + 231\,787\,008\, u\, z[u]^{25} -$$
$$80\,621\,568\, u^2\, z[u]^{25} - 117\,573\,120\, z[u]^{26} + 188\,116\,992\, u\, z[u]^{26} - 73\,903\,104\, u^2\, z[u]^{26} +$$
$$1\,679\,616\, u^3\, z[u]^{26} - 82\,301\,184\, z[u]^{27} + 151\,165\,440\, u\, z[u]^{27} - 75\,582\,720\, u^2\, z[u]^{27} +$$
$$6\,718\,464\, u^3\, z[u]^{27} - 57\,106\,944\, z[u]^{28} + 120\,932\,352\, u\, z[u]^{28} - 75\,582\,720\, u^2\, z[u]^{28} +$$
$$11\,757\,312\, u^3\, z[u]^{28} - 33\,592\,320\, z[u]^{29} + 80\,621\,568\, u\, z[u]^{29} - 60\,466\,176\, u^2\, z[u]^{29} +$$
$$13\,436\,928\, u^3\, z[u]^{29} - 16\,796\,160\, z[u]^{30} + 45\,349\,632\, u\, z[u]^{30} - 40\,310\,784\, u^2\, z[u]^{30} +$$
$$11\,757\,312\, u^3\, z[u]^{30} - 6\,718\,464\, z[u]^{31} + 20\,155\,392\, u\, z[u]^{31} - 20\,155\,392\, u^2\, z[u]^{31} +$$
$$6\,718\,464\, u^3\, z[u]^{31} - 1\,679\,616\, z[u]^{32} + 5\,038\,848\, u\, z[u]^{32} - 5\,038\,848\, u^2\, z[u]^{32} + 1\,679\,616\, u^3\, z[u]^{32}$$

$35\,831\,808\,z[u]^{16} + 73\,903\,104\,z[u]^{17} + 80\,621\,568\,z[u]^{18} + 112\,814\,208\,z[u]^{19} + 136\,608\,768\,z[u]^{20} +$
$169\,641\,216\,z[u]^{21} - 26\,873\,856\,u\,z[u]^{21} + 241\,864\,704\,z[u]^{22} - 81\,461\,376\,u\,z[u]^{22} + 270\,418\,176\,z[u]^{23} -$
$119\,252\,736\,u\,z[u]^{23} + 256\,141\,440\,z[u]^{24} - 148\,646\,016\,u\,z[u]^{24} + 231\,787\,008\,z[u]^{25} -$
$161\,243\,136\,u\,z[u]^{25} + 188\,116\,992\,z[u]^{26} - 147\,806\,208\,u\,z[u]^{26} + 5\,038\,848\,u^2\,z[u]^{26} +$
$151\,165\,440\,z[u]^{27} - 151\,165\,440\,u\,z[u]^{27} + 20\,155\,392\,u^2\,z[u]^{27} + 120\,932\,352\,z[u]^{28} -$
$151\,165\,440\,u\,z[u]^{28} + 35\,271\,936\,u^2\,z[u]^{28} + 80\,621\,568\,z[u]^{29} - 120\,932\,352\,u\,z[u]^{29} +$
$40\,310\,784\,u^2\,z[u]^{29} + 45\,349\,632\,z[u]^{30} - 80\,621\,568\,u\,z[u]^{30} + 35\,271\,936\,u^2\,z[u]^{30} + 20\,155\,392\,z[u]^{31} -$
$40\,310\,784\,u\,z[u]^{31} + 20\,155\,392\,u^2\,z[u]^{31} + 5\,038\,848\,z[u]^{32} - 10\,077\,696\,u\,z[u]^{32} + 5\,038\,848\,u^2\,z[u]^{32} -$
$350\,355\,456\,z[u]^{10}\,\text{Derivative}[1][z][u] - 418\,037\,760\,z[u]^{11}\,\text{Derivative}[1][z][u] -$
$181\,149\,696\,z[u]^{12}\,\text{Derivative}[1][z][u] + 463\,325\,184\,z[u]^{13}\,\text{Derivative}[1][z][u] +$
$1\,679\,616\,000\,z[u]^{14}\,\text{Derivative}[1][z][u] + 2\,394\,012\,672\,z[u]^{15}\,\text{Derivative}[1][z][u] +$
$573\,308\,928\,u\,z[u]^{15}\,\text{Derivative}[1][z][u] + 2\,622\,160\,512\,z[u]^{16}\,\text{Derivative}[1][z][u] +$
$1\,256\,352\,768\,u\,z[u]^{16}\,\text{Derivative}[1][z][u] + 1\,267\,270\,272\,z[u]^{17}\,\text{Derivative}[1][z][u] +$
$1\,451\,188\,224\,u\,z[u]^{17}\,\text{Derivative}[1][z][u] - 691\,441\,920\,z[u]^{18}\,\text{Derivative}[1][z][u] +$
$2\,143\,469\,952\,u\,z[u]^{18}\,\text{Derivative}[1][z][u] - 2\,908\,535\,040\,z[u]^{19}\,\text{Derivative}[1][z][u] +$
$2\,732\,175\,360\,u\,z[u]^{19}\,\text{Derivative}[1][z][u] - 4\,902\,799\,104\,z[u]^{20}\,\text{Derivative}[1][z][u] +$
$3\,562\,465\,536\,u\,z[u]^{20}\,\text{Derivative}[1][z][u] - 282\,175\,488\,u^2\,z[u]^{20}\,\text{Derivative}[1][z][u] -$
$5\,533\,494\,912\,z[u]^{21}\,\text{Derivative}[1][z][u] + 5\,321\,023\,488\,u\,z[u]^{21}\,\text{Derivative}[1][z][u] -$
$896\,075\,136\,u^2\,z[u]^{21}\,\text{Derivative}[1][z][u] - 5\,504\,941\,440\,z[u]^{22}\,\text{Derivative}[1][z][u] +$
$6\,219\,618\,048\,u\,z[u]^{22}\,\text{Derivative}[1][z][u] - 1\,371\,406\,464\,u^2\,z[u]^{22}\,\text{Derivative}[1][z][u] -$
$4\,766\,750\,208\,z[u]^{23}\,\text{Derivative}[1][z][u] + 6\,147\,394\,560\,u\,z[u]^{23}\,\text{Derivative}[1][z][u] -$
$1\,783\,752\,192\,u^2\,z[u]^{23}\,\text{Derivative}[1][z][u] - 3\,947\,097\,600\,z[u]^{24}\,\text{Derivative}[1][z][u] +$
$5\,794\,675\,200\,u\,z[u]^{24}\,\text{Derivative}[1][z][u] - 2\,015\,539\,200\,u^2\,z[u]^{24}\,\text{Derivative}[1][z][u] -$
$3\,056\,901\,120\,z[u]^{25}\,\text{Derivative}[1][z][u] + 4\,891\,041\,792\,u\,z[u]^{25}\,\text{Derivative}[1][z][u] -$
$1\,921\,480\,704\,u^2\,z[u]^{25}\,\text{Derivative}[1][z][u] + 43\,670\,016\,u^3\,z[u]^{25}\,\text{Derivative}[1][z][u] -$
$2\,222\,131\,968\,z[u]^{26}\,\text{Derivative}[1][z][u] + 4\,081\,466\,880\,u\,z[u]^{26}\,\text{Derivative}[1][z][u] -$
$2\,040\,733\,440\,u^2\,z[u]^{26}\,\text{Derivative}[1][z][u] + 181\,398\,528\,u^3\,z[u]^{26}\,\text{Derivative}[1][z][u] -$
$1\,598\,994\,432\,z[u]^{27}\,\text{Derivative}[1][z][u] + 3\,386\,105\,856\,u\,z[u]^{27}\,\text{Derivative}[1][z][u] -$
$2\,116\,316\,160\,u^2\,z[u]^{27}\,\text{Derivative}[1][z][u] + 329\,204\,736\,u^3\,z[u]^{27}\,\text{Derivative}[1][z][u] -$
$974\,177\,280\,z[u]^{28}\,\text{Derivative}[1][z][u] + 2\,338\,025\,472\,u\,z[u]^{28}\,\text{Derivative}[1][z][u] -$
$1\,753\,519\,104\,u^2\,z[u]^{28}\,\text{Derivative}[1][z][u] + 389\,670\,912\,u^3\,z[u]^{28}\,\text{Derivative}[1][z][u] -$
$503\,884\,800\,z[u]^{29}\,\text{Derivative}[1][z][u] + 1\,360\,488\,960\,u\,z[u]^{29}\,\text{Derivative}[1][z][u] -$
$1\,209\,323\,520\,u^2\,z[u]^{29}\,\text{Derivative}[1][z][u] + 352\,719\,360\,u^3\,z[u]^{29}\,\text{Derivative}[1][z][u] -$
$208\,272\,384\,z[u]^{30}\,\text{Derivative}[1][z][u] + 624\,817\,152\,u\,z[u]^{30}\,\text{Derivative}[1][z][u] -$
$624\,817\,152\,u^2\,z[u]^{30}\,\text{Derivative}[1][z][u] + 208\,272\,384\,u^3\,z[u]^{30}\,\text{Derivative}[1][z][u] -$
$53\,747\,712\,z[u]^{31}\,\text{Derivative}[1][z][u] + 161\,243\,136\,u\,z[u]^{31}\,\text{Derivative}[1][z][u] -$
$161\,243\,136\,u^2\,z[u]^{31}\,\text{Derivative}[1][z][u] + 53\,747\,712\,u^3\,z[u]^{31}\,\text{Derivative}[1][z][u]$

$31\,104\, z[u]^{10}\, \big(162\, (-1+u)^2\, z[u]^{22} - 11\,264\, \text{Derivative}[1][z][u] - 13\,440\, z[u]\, \text{Derivative}[1][z][u] -$

$\quad 5824\, z[u]^2\, \text{Derivative}[1][z][u] + 14\,896\, z[u]^3\, \text{Derivative}[1][z][u] +$

$\quad 54\,000\, z[u]^4\, \text{Derivative}[1][z][u] + 72\, (1069 + 256\, u)\, z[u]^5\, \text{Derivative}[1][z][u] +$

$\quad 216\, (-1+u)^2\, z[u]^{21}\, (3 + 8\, (-1+u)\, \text{Derivative}[1][z][u]) +$

$\quad 54\, (-1+u)\, z[u]^{20}\, \big(3\, (-9+7\, u) + 124\, (-1+u)^2\, \text{Derivative}[1][z][u]\big) +$

$\quad 54\, z[u]^{18}\, \big(72 - 90\, u + 21\, u^2 + 116\, (-1+u)^2\, (-5+2\, u)\, \text{Derivative}[1][z][u]\big) +$

$\quad 9\, z[u]^8\, (288 + 247\, (-10 + 31\, u)\, \text{Derivative}[1][z][u]) +$

$\quad 9\, z[u]^6\, (128 + 17\, (551 + 264\, u)\, \text{Derivative}[1][z][u]) +$

$\quad 27\, z[u]^7\, (88 + 3\, (503 + 576\, u)\, \text{Derivative}[1][z][u]) +$

$\quad 9\, z[u]^9\, (403 + 10\, (-1039 + 976\, u)\, \text{Derivative}[1][z][u]) +$

$\quad 324\, (-1+u)\, z[u]^{19}\, \big(4\, (-2+u) + 5\, \big(10 - 17\, u + 7\, u^2\big)\, \text{Derivative}[1][z][u]\big) -$

$\quad 18\, z[u]^{10}\, \big(-244 + 63\, \big(139 - 101\, u + 8\, u^2\big)\, \text{Derivative}[1][z][u]\big) -$

$\quad 27\, z[u]^{14}\, \big(-305 + 177\, u + 100\, \big(47 - 69\, u + 24\, u^2\big)\, \text{Derivative}[1][z][u]\big) -$

$\quad 27\, z[u]^{12}\, \big(-288 + 97\, u + 23\, \big(285 - 322\, u + 71\, u^2\big)\, \text{Derivative}[1][z][u]\big) -$

$\quad 27\, z[u]^{11}\, \big(-202 + 32\, u + 11\, \big(599 - 576\, u + 97\, u^2\big)\, \text{Derivative}[1][z][u]\big) -$

$\quad 54\, z[u]^{13}\, \big(-161 + 71\, u + 6\, \big(473 - 610\, u + 177\, u^2\big)\, \text{Derivative}[1][z][u]\big) +$

$\quad 108\, z[u]^{15}\, \big(69 - 48\, u + 13\, \big(-70 + 112\, u - 44\, u^2 + u^3\big)\, \text{Derivative}[1][z][u]\big) +$

$\quad 54\, z[u]^{16}\, \big(112 - 88\, u + 3\, u^2 + 27\, \big(-49 + 90\, u - 45\, u^2 + 4\, u^3\big)\, \text{Derivative}[1][z][u]\big) +$

$\quad 108\, z[u]^{17}\, \big(45 - 45\, u + 6\, u^2 + 14\, \big(-34 + 72\, u - 45\, u^2 + 7\, u^3\big)\, \text{Derivative}[1][z][u]\big)\big)$

$\big\{\big\{\text{Derivative}[1][z][u] \rightarrow$

$\quad - \big(9\, \big(128\, z[u]^6 + 264\, z[u]^7 + 288\, z[u]^8 + 403\, z[u]^9 + 488\, z[u]^{10} + 606\, z[u]^{11} - 96\, u\, z[u]^{11} +$

$\qquad 864\, z[u]^{12} - 291\, u\, z[u]^{12} + 966\, z[u]^{13} - 426\, u\, z[u]^{13} + 915\, z[u]^{14} - 531\, u\, z[u]^{14} +$

$\qquad 828\, z[u]^{15} - 576\, u\, z[u]^{15} + 672\, z[u]^{16} - 528\, u\, z[u]^{16} + 18\, u^2\, z[u]^{16} + 540\, z[u]^{17} -$

$\qquad 540\, u\, z[u]^{17} + 72\, u^2\, z[u]^{17} + 432\, z[u]^{18} - 540\, u\, z[u]^{18} + 126\, u^2\, z[u]^{18} + 288\, z[u]^{19} -$

$\qquad 432\, u\, z[u]^{19} + 144\, u^2\, z[u]^{19} + 162\, z[u]^{20} - 288\, u\, z[u]^{20} + 126\, u^2\, z[u]^{20} +$

$\qquad 72\, z[u]^{21} - 144\, u\, z[u]^{21} + 72\, u^2\, z[u]^{21} + 18\, z[u]^{22} - 36\, u\, z[u]^{22} + 18\, u^2\, z[u]^{22}\big)\big) \,\Big/$

$\quad\big(-11\,264 - 13\,440\, z[u] - 5824\, z[u]^2 + 14\,896\, z[u]^3 + 54\,000\, z[u]^4 + 76\,968\, z[u]^5 + 18\,432\, u\, z[u]^5 +$

$\qquad 84\,303\, z[u]^6 + 40\,392\, u\, z[u]^6 + 40\,743\, z[u]^7 + 46\,656\, u\, z[u]^7 - 22\,230\, z[u]^8 + 68\,913\, u\, z[u]^8 -$

$\qquad 93\,510\, z[u]^9 + 87\,840\, u\, z[u]^9 - 157\,626\, z[u]^{10} + 114\,534\, u\, z[u]^{10} - 9072\, u^2\, z[u]^{10} -$

$\qquad 177\,903\, z[u]^{11} + 171\,072\, u\, z[u]^{11} - 28\,809\, u^2\, z[u]^{11} - 176\,985\, z[u]^{12} + 199\,962\, u\, z[u]^{12} -$

$\qquad 44\,091\, u^2\, z[u]^{12} - 153\,252\, z[u]^{13} + 197\,640\, u\, z[u]^{13} - 57\,348\, u^2\, z[u]^{13} - 126\,900\, z[u]^{14} +$

$\qquad 186\,300\, u\, z[u]^{14} - 64\,800\, u^2\, z[u]^{14} - 98\,280\, z[u]^{15} + 157\,248\, u\, z[u]^{15} - 61\,776\, u^2\, z[u]^{15} +$

$\qquad 1404\, u^3\, z[u]^{15} - 71\,442\, z[u]^{16} + 131\,220\, u\, z[u]^{16} - 65\,610\, u^2\, z[u]^{16} + 5832\, u^3\, z[u]^{16} -$

$\qquad 51\,408\, z[u]^{17} + 108\,864\, u\, z[u]^{17} - 68\,040\, u^2\, z[u]^{17} + 10\,584\, u^3\, z[u]^{17} - 31\,320\, z[u]^{18} +$

$\qquad 75\,168\, u\, z[u]^{18} - 56\,376\, u^2\, z[u]^{18} + 12\,528\, u^3\, z[u]^{18} - 16\,200\, z[u]^{19} + 43\,740\, u\, z[u]^{19} -$

$\qquad 38\,880\, u^2\, z[u]^{19} + 11\,340\, u^3\, z[u]^{19} - 6696\, z[u]^{20} + 20\,088\, u\, z[u]^{20} - 20\,088\, u^2\, z[u]^{20} +$

$\qquad 6696\, u^3\, z[u]^{20} - 1728\, z[u]^{21} + 5184\, u\, z[u]^{21} - 5184\, u^2\, z[u]^{21} + 1728\, u^3\, z[u]^{21}\big)\big\}\big\}$

$\quad - 0.0412858$

According to Drmota's Theorem 1, the mean equals -z'[1]/z[1], computed next.

$\quad 0.0669564$

$\quad 0.0322769$

Now compute variance, which by Drmota is -z''[1]/z[1] + mu^2 + mu

$\quad 0.0190935$