In[121]:= **(* Fusy−Zuker grammar *)**

```
Clear[S, S0, P, P0, Q, rho, growth, Gnum, Gdenom, B, M, M1, U, z];

(* DSV equations from Zuker's treatment of dangles*)
eqn = {S == z + S z + (B + S B) (1 + 2 z + z^2),
   B == U z^2 + B z^2 + 2 (z + z U + U) B z^2 + (z + z U + U) ^2 B z^2 + M M1 z^2 ,
   M == M1 (1 + U + M), M1 == M1 z + B (1 + 2 z + z^2), U == z + U z};
Solve[eqn, {S, B, M, M1, U}];

(* S0 is solution for S the generating function for S *)
```

In[164]:=

$$\mathrm{S0} = -\frac{1}{z^2}\left(-\frac{1}{2\left(1+2z+z^2\right)}+\frac{z}{2\left(1+2z+z^2\right)}+\frac{z^2}{2\left(1+2z+z^2\right)}+\frac{z^3}{1+2z+z^2}+\frac{z^4}{2\left(1+2z+z^2\right)}+\left(\sqrt{\left(-4z^3\left(1+2z+z^2\right)+\left(-1+z+z^2+2z^3+z^4\right)^2\right)}\right)\Big/\left(2\left(1+2z+z^2\right)\right)\right);$$

```
(* Simplify S0 into expression S1 *)
Simplify[S0];

(* Now S=S0=G+H, where G=Gnum/Gdenom and H=-sqrt(P)/Gdenom*)
```

$\mathrm{Gnum} = -\left(-1+z+z^2+2z^3+z^4\right);$

$\mathrm{Gdenom} = \left(2z^2\,(1+z)^2\right);$

`G = Gnum / Gdenom;`

$P = 1-2z-z^2-6z^3-5z^4+2z^5+6z^6+4z^7+z^8;$

```
(* P0 is P*)
P0 = P;
(* Cannot symbolically solve P1==0; however get numerical solution *)
NSolve[{P0 == 0}, z];


(* dominant singularity ie smallest modulus root of eqn1 *)
rho = 0.32471795724474606`;


(* growth base of exponential is 3.0796`.
 For some reason 1/rho doesn't replace rho by value.*)
growth = N[1 / 0.32471795724474606`];
```

In[177]:=

```
(* Now rho is dominant singularity.Want to factor (1-z/rho) from P. This is done by
  computing Q=Series[P/(1-z/rho),{z,0,20}] and taking the resulting polynomial. *)
TaylorExpansionOfQ = Series[P / (1 - z / 0.32471795724474606`), {z, 0, 20}];

(* P=(1-z/rho)*Q; ie Q = P/(1-z/rho)  *)
```

$Q = \{1 + 1.0795956234914383\text{`}\, z + 2.3247179572447436\text{`}\, z^2 + 1.1591912469828678\text{`}\, z^3 - 1.4301597090019698\text{`}\, z^4 - 2.4043135807362432\text{`}\, z^5 - 1.404313580736357\text{`}\, z^6 - 0.3247179572454115\text{`}\, z^7\};$

In[179]:=

```
(* Now S=
 G+H and H=-sqrt(1-z/rho)*sqrt(Q)/Gdenom. We now evaluate G(rho) to obtain G(rho)=
   1.3247179572447456` *)
G /. (z → 0.32471795724474606`);
```

```
(* Evaluate Gdenom(rho) = 0.3700747504972791` *)
Gdenom /. {z → 0.32471795724474606`}

(* Now evaluate Q at rho,
 i.e. Q/.{z→0.31335475182708045`}. We get Q(rho) = 0.3700747504972791`.} *)
{1 + 1.0795956234914383` z + 2.3247179572447436` z^2 + 1.1591912469828678` z^3 -
    1.4301597090019698` z^4 - 2.4043135807362432` z^5 - 1.404313580736357` z^6 -
    0.3247179572454115` z^7} /. {z → 0.32471795724474606`};
```

Out[182]= 0.370075

Out[183]= {1.60903}

```
(* Now evaluate Sqrt[Q(rho)] to get 1.268473637159293` *)
```

In[186]:=
```
Sqrt[1.6090253681681257`];

(* It follows that S(rho) sim G(rho)-sqrt(1-z/rho)*sqrt(Q(rho))/Gdenom(rho) *)
Sfinal = 1.268473637159293` -
  (Sqrt[1 - z / 0.32471795724474606`] * 1.268473637159293`) / 0.3700747504972791`
```

Out[187]= $1.26847 - 3.42761 \sqrt{1 - 3.0796\, z}$

```
(* It follows that S(rho) sim G(rho)-K sqrt(1-z/rho),
where G(rho) =1.268473637159293`` and K =3.4276146520528945` *)

(* Gamma(-1/2) = -2 sqrt(pi) *)

Gamma[-1 / 2];

(* -K/Gamma(-1/2) = 0.9669122415519551` *)
```

$(-3.4276146520528945\text{`}) \Big/ \left(-2 \sqrt{\pi}\right);$

Out[188]= 0.966912

```
(* Asymptotic solution *)
(* It follows that S_n sim 0.9669122415519551` * n^{-3/2} * (3.0795956234914383`)^n *)
```