```
(*==========================================
  ================
 Saturated secondary structures
       computed by DSV and Flajolet–
      Odlyzko. Context free grammar given by

       s ––> * or ** or *R or **R or (S) or S(S)
      R ––> (S) or R(S)

    ==================================
    ====================*)

P = Simplify[Resultant[-S + z  + z^2 +
     z R + z^2 R + z^2 S + z^2 S^2, -R + z^2 S + z^2 R S, R]]
DP = Simplify[D[P, S]]
Factor[
 Resultant[-S^3 z^4 + z (1 + z) - S^2 z^2 (-2 + z^2) + S (-1 + z^2),
  -1 + (1 + 4 S) z^2 - S (2 + 3 S) z^4, S]]
```

$$-1 + (1 + 4\,S)\,z^2 - S\,(2 + 3\,S)\,z^4$$

$$z^{11}\,(1 + z)\,\left(4 + z - 7\,z^2 - 28\,z^3 - 32\,z^4 + 4\,z^6\right)$$

$$-S^3\,z^4 + z\,(1 + z) - S^2\,z^2\,\left(-2 + z^2\right) + S\,\left(-1 + z^2\right)$$

```
(* Find dominant singularity rho *)

eqn = {4 + z - 7 z^2 - 28 z^3 - 32 z^4 + 4 z^6 == 0}
NSolve[eqn, z]
```

$$\left\{4 + z - 7\,z^2 - 28\,z^3 - 32\,z^4 + 4\,z^6 == 0\right\}$$

```
{{z → -2.29493}, {z → -0.854537}, {z → -0.244657 - 0.5601 i},
 {z → -0.244657 + 0.5601 i}, {z → 0.424687}, {z → 3.2141}}
```

```
rho = 0.4246873104202721`

F = 4 + z - 7 z^2 - 28 z^3 - 32 z^4 + 4 z^6
growth = 1 / rho
Q = Series[F / (1 - z / rho), {z, 0, 8}]
Expand[Series[Q, {z, 0, 8}] * (1 - z / rho)]
```

```
0.424687
```

$$0.605047 - 0.819641\,S + 0.328189\,S^2 - 0.0325295\,S^3$$

```
DP
```

$$4 + z - 7\,z^2 - 28\,z^3 - 32\,z^4 + 4\,z^6$$

```
2.35467
```

$$4 + 10.4187\,z + 17.5326\,z^2 + 13.2836\,z^3 - 0.721437\,z^4 - 1.69875\,z^5 + 2.27374\times10^{-13}\,z^6 + 4.54747\times10^{-13}\,z^7 + 4.54747\times10^{-13}\,z^8 + O[z]^9$$

$$4 + z - 7.\,z^2 - 28.\,z^3 - 32.\,z^4 - 6.88338\times10^{-15}\,z^5 + 4.\,z^6 - 8.06434\times10^{-14}\,z^7 - 6.16034\times10^{-13}\,z^8 + O[z]^9$$

```
(*=============================================================
  ========
    Compute value T and verify that
      dP/dS is increasing when z is rho and S is T. Find possible
      values of T by solving equation in S,
and then discarding the possible root TT below.
  =============================================================
  ===*)
```

```
NSolve[0.6050466220522767` - 0.8196406883679954` S +
    0.3281891419716385` S^2 - 0.032529481292370 52` S^3 == 0, S]
T = 1.6568963314781193`
Simplify[Series[P, {z, rho, 4}, {S, T, 4}]]
epsilon = 0.001
TT = 6.7751828754381425`
DP /. {z → rho, S → TT + epsilon}
Plot3D[DP, {z, rho - epsilon, rho + epsilon},
  {S, T - epsilon, T + epsilon}];
```

$$-S^3 z^4 + z\,(1+z) - S^2 z^2\left(-2+z^2\right) + S\left(-1+z^2\right)$$

0.424687

$$0.605047 - 0.819641\,S + 0.328189\,S^2 - 0.0325295\,S^3$$

$$\{\{S \to 1.6569\},\ \{S \to 1.6569\},\ \{S \to 6.77518\}\}$$

1.6569

$$\left(-4.79202\times 10^{-9}\,(S-1.6569) + 0.166495\,(S-1.6569)^2 - 0.0325295\,(S-1.6569)^3 + O[S-1.6569]^5\right) +$$
$$\left(5.68551 + 2.94002\,(S-1.6569) - 0.130582\,(S-1.6569)^2 - 0.306385\,(S-1.6569)^3 + O[S-1.6569]^5\right)$$
$$(z-0.424687) +$$
$$\left(0.254271 - 4.871\,(S-1.6569) - 4.46122\,(S-1.6569)^2 - 1.08216\,(S-1.6569)^3 + O[S-1.6569]^5\right)$$
$$(z-0.424687)^2 +$$
$$\left(-12.3907 - 19.6201\,(S-1.6569) - 10.1427\,(S-1.6569)^2 - 1.69875\,(S-1.6569)^3 + O[S-1.6569]^5\right)$$
$$(z-0.424687)^3 +$$
$$\left(-7.29399 - 11.5497\,(S-1.6569) - 5.97069\,(S-1.6569)^2 - (S-1.6569)^3 + O[S-1.6569]^5\right)$$
$$(z-0.424687)^4 + O[z-0.424687]^5$$

0.001

6.77518

DP

```
DPz = D[P, z]
```

$$1 + 2z + 2Sz - 2S^2 z^3 - 4S^3 z^3 - 2S^2 z\left(-2+z^2\right)$$

```
DPS2 = D[DP, S]
```

$$1 + 2z + 2Sz - 2S^2 z^3 - 4S^3 z^3 - 2S^2 z\left(-2+z^2\right)$$

$$4z^2 - 3Sz^4 - (2+3S)\,z^4$$

```
C1 = DPz /. {z → rho, S → T}
```

```
5.68551
```

```
C2 = DPS2 /. {z → rho, S → T}
```

```
K = Sqrt[2 rho C1 / C2]
```

```
3.808190403387938`
K / (2 Sqrt[Pi])
Gamma[-1 / 2]
```

```
3.80819
```

```
1.07427
```

$$-2\sqrt{\pi}$$

```
SatSecStr[n_] = 1.0742706788790013` n^(-3 / 2) rho^(-n)
```

$$-2\sqrt{\pi}$$

$$\frac{1.0742706788790013` \, 0.4246873104202721`^{-n}}{n^{3/2}}$$

```
721655.
```